Universitat Autònoma de Barcelona

Departament d'Economia i d'Història Econòmica
International Doctorate in Economic Analysis (IDEA)

Essays on Money and Endogenous Growth

Doctoral Dissertation

Jana Hromcová

Advisor: Jordi Caballé

Bellaterra, July 1999

Acknowledgments

At this place I would like to thank all who contributed to this thesis by their discussions, comments and suggestions or by simply being where I needed them to be.

In the first place I want to thank Jordi Caballé for giving me the opportunity to work with him and learn so much from him, for his professional guidance, for being so demanding but also very patient with me and for making me always believe that I can make it. También quiero agradecer a Hugo Rodríguez Mendizábal por sus comentarios y sugerencias que me llevaron a ensanchar mis horizontes, sobre todo en lo concerniente al modelo del segundo capítulo y por animarme a entrar en el mundo de las expectativas parametrizadas, a Josefina Monteagudo por sus comentarios y ánimos, a Jesús Ruiz por aclararme cualquier duda con respecto a las simulaciones numéricas, y a Omar Licandro que en la fase final me hizo mirar algunos aspectos de mi trabajo desde otro ángulo.

Gràcies al Departament d'Economia i d'Història Econòmica per donar-me l'oportunitat per fer la recerca aquí y per poder passar aquest cinc anys a la UAB i per conèixer aquesta cuitat tan meravellosa.

I am grateful for helpful comments from the participants of the Macroeconomics Workshops at Universitat Autònoma de Barcelona and Universitat Pompeu Fabra.

Luego quiero dar las gracias a los amigos que hice durante estos años en Barcelona, los cuales compartieron conmigo los malos y los buenos momentos. Tiež chcem poďakovať mojim kamarátom v Bratislave, ktorí i keď nie sú tak blízko, stále sú so mnou v kontakte a podporujú ma. En particular quiero mencionar a Laura, por escucharme y discutir toda clase de problemas, a Josep, por ser tan especial, estar tan cerca y actuar para mi de 'madrimoni', a na tomto mieste nemôžem nespomenúť Ozzyka, ktorý je 'najvernejší' a stále tak blízko.

Ďalej chcem poďakovať mojej rodine, mame a oťovi, babinke a dedkovi, Kukovi a Karinkovi, za to že sú trpezliví, a i keď so mnou nie vždy súhlasia, i tak ma neustále podporujú. I finalment a Mateu, per tot, sobre tot per sa seva paciència i per ensenyar-me que "if there's a will, there's a way".

Contents

Chapter 1

Introduction

The modern literature on money and growth begins with Tobin (1965) who addresses a question which has been studied by many authors since then: what are the real effects of the growth rate of the money supply? Tobin finds that a faster money growth is associated with a higher capital stock and a higher level of output per capita. This is so because a higher rate of monetary growth creates inflation and this reduces the return on money so that productive investment becomes more profitable.

Sidrauski (1967) overturns the results of Tobin by introducing the real monetary balances in the utility function. In his model the long run stock of capital is independent of the money growth rate. In many other works the superneutrality result is confirmed. For example, Danthine, Donaldson and Smith (1987) present a stochastic model with money in the utility function in which money come very close to being superneutral.

In other models, high inflation decreases both capital and output so neither Tobin's, nor Sidrauski's results apply. In particular, inflation acts as a tax on productive activity and therefore retards capital accumulation in the cash-in-advance models studied by Stockman (1981) or Cooley and Hansen (1989, 1991).[1]

Looking at the growth side, the papers of Romer (1986), Lucas (1988) and Rebelo (1991) represent the initial wave of modern research in growth theory. Sustained growth is achieved because returns to investment do not necessarily diminish as economies develop. A constant returns to scale production function at the aggregate level can be achieved by spillovers of knowledge, by public goods provided by the government, or

[1] Wang and Yip (1992) provide a review of models analyzing the question of money and growth using three approaches: money in the utility function, cash-in-advance and transaction costs models. An extensive survey on money, inflation and growth can be also found in Orphanides and Sollow (1990).

by one-sector models with physical and human capital, as shown in Barro and Sala-i-Martin (1995). Most of the critics to such models argue that the absence of diminishing returns to scale has little empirical support. Without diminishing returns, a country with a high stock of capital is not deterred from continued investment, and therefore from continued growth. McGrattan (1998) shows that taking into account data for a long period it can be found that the key prediction of these models holds, namely, that periods of high investment rates coincide roughly with periods of high growth rates. Even more, if the convergence occurs slowly, as seems to be the case, the growth effects that appear in such models provide a satisfactory approximation to the effects on the average growth rate over a long interval in the neoclassical model.

The topic of this dissertation is also closely related to the business cycle literature. Modern equilibrium business cycle theory goes back to the works of Lucas (1972, 1975). Lucas results were only qualitative in nature. Later, Kydland and Prescott (1982) develope the methodology for obtaining quantitative results for equilibrium business cycle models and their techniques are now standard in the theoretical literature on aggregate fluctuations. Kydland and Prescott (1982) only analyze a model in which the impulses that lead to business cycles are shocks to technology. Cooley and Hansen (1989, 1995) incorporate money into business cycle models and examine the importance of the monetary shocks in fluctuations of real variables. They find that when the money supply growth rate is fixed, the performance of the business cycle is not affected. On the other hand, when money is supplied erratically, the characteristics of the business cycle are somewhat altered.

We are interested in the cash-in-advance constraint as a device of introducing money into macroeconomic models. Bohn (1991) argues that cash-in-advance models can yield a money demand function consistent with the empirical literature. What we basically require of monetary models is that money is demanded even when its return is dominated, money demand is positively related to aggregate output or consumption, income velocity is variable, and velocity is positively related to the interest rates on other assets. These features generally hold introducing uncertainty or cash and credit goods. Among models that meet the required features are, for example, Lucas (1984), Lucas and Stokey (1983, 1987), Ireland (1994), Jones and Manuelli (1995).

One of the features which is to be met in models with money and which is sometimes neglected is a variable income velocity. Fluctuations of money velocity are mostly explained by the reaction of real balances to changes in nominal interest rates. An increasing trend in velocity is usually attributed to an increasing trend in inflation and nominal interest rates, to a decline in the use of money as a medium of exchange when the use of alternative means of transactions, as higher availability of credit arrangements, increases, and to the increased frequency of asset market transactions that allows households and firms to hold fewer real balances. More frequent transactions are explained by the development of financial markets that has lowered the transaction costs. An extensive survey on this topic can be found in Cole and Ohanian (1997) and Gordon, Leeper and Zha (1998).

Throughout this dissertation we present models in which the variability of the income velocity arises due to different effects. Each chapter presents a different way of modeling variable income velocity. In Chapter 2 variability of money velocity is due to changes in nominal interest rates and to the uncertainty about the realization of the state of the economy. In Chapter 3 income velocity varies due to shocks in technology and a trend in velocity may arise due to a decline in the use of money as a medium of exchange. Such a decline is due to the fact that an alternative mean of payment becomes relatively cheaper in the course of time. In Chapter 4 shocks to technology and in the efficiency of the payment system induce a variation into the demand for real balances, and therefore, income velocity also fluctuates. Of course trends in velocity may appear when the efficiency of the payment system increases in the course of time.

Concerning money and growth, one of the interesting questions to analyze is: what are the effects of economic growth on the monetary system, and which factors influence them? As mentioned in the first paragraph of this introduction, money-growth relationship was extensively explored in the literature by taking into account the influence of money on growth, that is, by looking for an answer to the question already posed by Tobin (1965). Ireland (1994) studies the money-growth relationship from both sides. He studies what are the effects of inflation on growth and how growth influences the existing monetary system. He finds that the traditional effects of inflation on growth are small and the monetary system changes significantly due to economic growth. We will devote one chapter of this thesis to the alternative analysis of the money-growth relationship and we will observe the effects of growth on the monetary system and show that this relationship is closely related to the behavior of financial intermediaries.

Since we deal with monetary models, we must pay an appropriate attention to the monetary policy. In fact, one of the oldest debates in monetary economics concerns the choice of the suitable target for monetary policy. Taking into account that monetary authorities may control either monetary aggregates or nominal interest rates, but not both independently, a natural question arises: should central banks target the money supply growth rate or the nominal interest rates? In the last decades many central banks have reoriented their operating procedures to focus more on the nominal interest rate targeting and less on the monetary aggregate targeting. The reasons may be in part based on the theoretical results by Poole (1970) who suggests that a central bank would have a better control of the price level if it targeted the nominal interest rates instead of the monetary aggregates. However, the effects on the output stability, growth and welfare are not so clear. Canzoneri and Dellas (1998) find that the nominal interest rate targeting stabilizes prices, but produces higher real interest rates. Collard, Dellas and Ertz (1998) find that targeting the nominal interest rate is a price and output stabilizing policy, and leads also to higher welfare. Carlstrom and Fuerst (1995) find that the nominal interest rate targeting leads to higher volatility of output than the monetary aggregate targeting but it delivers higher welfare. Recently, some alternative targets as the nominal income or the (expected) inflation targeting have become popular. For example, it seems that the first three inflation targeters (the central banks of New Zealand, Canada and Britain) achieved price stability without penalizing growth by switching away from monetary aggregate targeting, as reported in The Economist (1997). We make our contribution to this debate in Chapter 4.

In the first part of this dissertation (Chapter 2) we evaluate the changes in the income velocity of money that arise due to a precautionary money demand. We extend the analysis of Svensson (1985) to a model with endogenous growth. We characterize under which conditions a precautionary money demand occurs. We evaluate how much variability is introduced by a precautionary money demand into the income velocity of money.

The main findings are the following. As the only way to consume is using money, agents make a precautionary money demand in some states of the nature because of the uncertainty about the realization of the state of the economy. Individuals who want to smooth consumption over periods might make a precautionary money demand in bad times. Unspent balances can be saved in the future period in the form of capital. Higher capital accumulation accelerates growth and the effects of low shocks on consumption can be thus slightly dampened. The fact that agents make a precautionary money demand would contribute to a 2-10% decrease in the average income velocity if they made it in almost all states of the world. Such a situation would occur if the average levels of the money growth rate were quite low compared to the one in the US data (which are the data used to calibrate the model). Therefore, we conclude that the mechanism of a precautionary money demand does not lead to quantitatively important changes in the income velocity of money.

In Chapter 3 we study how the specification of the payment that intermediaries charge for providing financial services affects the relationship between growth and the monetary system. In our model agents face a choice when they purchase consumption goods: they can use either money or the services of an intermediary. When the individuals consume via services of an intermediary they have to pay a cost which is a function of the purchases acquired via the intermediary (which can be viewed as a cost to be paid for the availability of an alternative mean of payment). We build on works of Gillman (1993) and Aiyagari, Braun and Eckstein (1995) who formalize the idea that monetary policy influences the decisions to devote resources to the creation of money substitutes. Similarly as Ireland (1994) we let growth be a factor that may increase the demand for alternative means of payment.

We show that when the intermediation cost increases proportionally to the purchases via an intermediary, we do not observe any influence of growth on the monetary system. When the intermediation cost is a non-proportional function of purchases via intermediary, growth yields a transformation of the monetary system, i.e., money is relatively driven out of the economy as the economy grows. Agents consume using money always when the services of intermediary are relatively expensive. As the economy gets richer when growing, it becomes relatively cheaper to employ services of intermediary and agents switch away from using cash to consume via money substitutes. Finally, higher nominal interest rates accelerate the transformation of the economy.

In Chapter 4 of this dissertation we compare the performance of an economy with productive government spending under two central bank operating procedures. That is, we let the monetary authorities conduct the monetary policy in two ways: regulating monetary aggregates or nominal interest rates. As other studies suggest, alternative monetary policy targets may lead to different fluctuations in output and prices and to different levels of welfare.

Related issues were analyzed for example by Poole (1970) in an IS-LM model, by Carlstrom and Fuerst (1995) in a cash-in-advance model with portfolio rigidity in the households' cash savings choice, by Canzoneri and Dellas (1998) in a model with and without labor contracts inducing rigidities, and by Collard, Dellas and Ertz (1998) in an exogenous growth model. We reexamine the issues of the choice of monetary policy targets in an endogenous growth model, where the government finances its spending by taxes and seignorage.

We allow for two kinds of shocks in our model, technology and money demand shocks (which are also referred to as shocks in the efficiency of the payment system). Whenever the efficiency of the payment system increases, money has less value and the demand for real balances decreases. The corresponding changes in prices influence in a significant way the evolution of the other variables in the economy.

We find that output is less volatile under the monetary aggregate targeting, regardless of the origin of disturbances. Poole (1970) arrives to the same result when the origin of disturbances are the shocks in the money demand, but not when the fluctuations come from shocks in technology. Concerning the inflation rate, our results confirm the findings of Poole (1970), Canzoneri and Dellas (1998) and Collard, Dellas and Ertz (1998) that the inflation rate is less volatile under the interest rate targeting. Concerning welfare, we find that none of the analyzed targeting procedures is clearly superior, even if the volatility of consumption is lower under the nominal interest rate targeting.

Studying stochastic growth models is not always possible using analytical tools, especially when correlated shocks are considered. Advances in dynamic economic theory and progress in computational methods over the past two decades have provided new tools to treat such issues. In fact there are only some combinations of utility and production functions that enable analytical solutions, like for instance the ones in Sargent (1987), McCallum (1989), Stokey, Lucas and Prescott (1989) and Hercowitz and Sampson (1991).

There have been developed many numerical algorithms to solve stochastic growth models. Approaches to solve linear systems can be found in Hansen and Sargent (1996). Considering non-linear models, there exists several articles that compare the performance of the most widely used numerical techniques. Let us mention the article of Taylor and Uhlig (1990) that compares the performance of several commonly used techniques applied to a simple neoclassical stochastic growth model. Santos (1998) also compares some of the numerical techniques paying special attention to their accuracy and stability properties by considering an application to a neoclassical growth model with labor. An application of the log-linear method due to Uhlig (1997) and the parameterized expectations method due to Den Haan and Marcet (1990) to a two sector endogenous growth model with human capital is presented in Barañano, Iza and Vázquez (1997). These authors evaluate the performance of the two techniques in terms of accuracy and computing time. It seems to be the case that the parameterized expectations method outperforms the log-linear technique in terms of precision. However, Santos (1998) argues that the performance of a particular numerical algorithm depends mainly on the model it is applied to.

Den Haan and Marcet (1994) suggest a test where the Euler equation residuals are studied in order to check for the accuracy. Such a test statistic is analogous to the one used to check for overidentifying restrictions in the generalized method of moments estimation originally suggested in Hansen (1982).

Models with money can be solved using analogous approaches. Examples of numerical techniques to solve monetary stochastic growth models can be found for instance in the works of Cooley and Hansen (1989, 1995) and Hansen and Prescott (1995). The literature does not report on any comparison of the performance of different techniques in endogenous growth models with money. However, following the stream of articles of Den Haan (1990), Rodríguez-Mendizábal (1997), Santos (1998) it seems that the method of parameterized expectations is quite appropriate.

For our application we choose two out of the numerous techniques: the log-linear method of Uhlig (1997) and the method of parameterized expectations of Den Haan and Marcet (1990). Uhlig simplifies and unifies existing techniques and his algorithm is easy to apply. It is a method based on the log-linearization of the necessary equations characterizing the equilibrium. It allows one to solve for the recursive equilibrium law of motion using the method of undetermined coefficients. The algorithm is an Euler equation based approach and the solution of non-optimal problems does not pose any additional difficulty. The method of parameterized expectations is also an Euler equation based technique. It consists of approximating the conditional expectations appearing in the Euler equations by a functional form which is dependent on the state of the system, and estimating of the parameters of that function in a particular model. The advantage of the log-linear method is that, once the necessary information is plugged into the computer, the results are obtained immediately. On the other hand, the method of parameterized expectations is much more demanding in terms of computing time. However, its main advantage is that the non-linear system does not have to be linearized, and thus the solution is more precise. Moreover, the parameterized expectation algorithm can be applied to a more general class of models.

Bibliography

[1] Aiyagari, S. Rao, Braun, Toni and Eckstein, Zvi, 1995, "Transaction Services, Inflation, and Welfare", Federal Reserve Bank of Minneapolis, Research Department, Working Paper 551

[2] Barañano, Ilaski, Iza, Amaia and Vázquez, Jesús, 1997 "A Comparison Between the Parameterized Expectations and Log-linear Simulation Methods", Universidad del País Vasco, mimeo

[3] Barro, J. Robert and Sala-i-Martin, Xavier, 1995, Economic Growth, McGraw-Hill

[4] Bohn, Henning, 1991,"On Cash-in-Advance Models of Money Demand and Asset Pricing", *Journal of Money, Credit and Banking,* Vol. 23, No. 2, pp. 224-242

[5] Canzoneri, Matthew B. and Dellas, Harris, 1998, "Real Interest and Central Bank Operating Procedures", *Journal of Monetary Economics,* Vol. 42, pp. 471-494

[6] Carlstrom, Charles T. and Fuerst, Timothy S., 1995, "Interest Rate Rules vs. Money Growth Rules A Welfare Comparison in a Cash-in-Advance Economy", *Journal of Monetary Economics,* Vol. 36, pp. 247-267

[7] Cole, Harold L. and Ohanian, Lee E., 1997, "Shrinking Money and Monetary Business Cycles", Federal Reserve Bank of Minneapolis, Research Department Working Paper 579

[8] Collard, Fabrice, Dellas, Harris and Ertz, Guy, 1998, "Poole Revisited", IRES, Université Catolique de Louvain, Discussion Paper 9823

[9] Cooley, Thomas F. and Hansen, Garry D., 1989, "The Inflation Tax in Real Business Cycle Model", *The American Economic Review,* Vol. 79, No. 4, pp. 733-748

[10] Cooley, Thomas F. and Hansen, Garry D., 1991, "The Welfare Costs of Moderate Inflations" *Journal of Money, Credit, and Banking,* Vol. 23, pp. 483-503

[11] Cooley, Thomas F. and Hansen, Garry D., 1995, "Money and the Business Cycle" in Frontiers of Business Cycle Research, Ed. Thomas F. Cooley, Princeton University Press

[12] Danthine, Jean-Pierre, Donaldson, John B. and Smith, Lance, 1987, "On the Superneutrality of Money in a Stochastic Dynamic Macroeconomic Model", *Journal of Monetary Economics,* Vol. 20, pp. 475-499

[13] Den Haan, Wouter J., 1990, "The Optimal Inflation Path in a Sidrauski-Type Model with Uncertainty", *Journal of Monetary Economics,* Vol. 25, pp. 389-409

[14] Den Haan, Wouter J. and Marcet, Albert, 1990, "Solving the Stochastic Growth Model by Parameterizing Expectations", *Journal of Business and Economic Statistics,* Vol. 8, No. 1, pp. 31-34

[15] Den Haan, Wouter J. and Marcet, Albert, 1994, "Accuracy in Simulations", *Review of Economic Studies,* Vol. 61, pp. 3-17

[16] Economist, The, 1997, September 6th, Economic Focus, p. 83

[17] Gillman, Max, 1993, "The Welfare Cost of Inflation in a Cash-in-Advance Economy with Costly Credit", *Journal of Monetary Economics,* Vol. 32, pp. 51-77

[18] Gordon, David B., Leeper Eric M. and Zha, Tao, 1998, "Trends in Velocity and Policy Expectations", *Carnegie-Rochester Conference Series on Public Policy,* Vol. 49, pp. 265-304

[19] Hansen, Lars Peter, 1982, "Large Sample Properties of Generalized Method of Moments Estimators", *Econometrica,* Vol. 50, No. 4, pp. 1029-1054

[20] Hansen, Garry D. and Prescott, Edward C., 1995, "Recursive Methods for Computing Equilibria of Business Cycle Models" in Frontiers of Business Cycle Research, Ed. Thomas F. Cooley, Princeton University Press

[21] Hansen, Lars Peter and Sargent, Thomas J., 1996, Recursive Linear Models of Dynamic Economies, http://riffle.stanford.edu/hansen.html

[22] Hercowitz, Zvi and Sampson, Michael, 1991, "Output Growth, the Real Wage, and Employment Fluctuations", *The American Economic Review,* Vol. 81, No. 5, pp. 1225-1237

[23] Ireland, Peter N., 1994, "Money and Growth: An Alternative Approach", *The American Economic Review,* Vol. 84, No. 1, pp. 47-65

[24] Jones, Larry E. and Manuelli, Rodolfo E. ,1995, "Growth and the Effects of Inflation", *Journal of Economic Dynamics and Control,* Vol. 19, pp. 1405-1428

[25] Kydland, F. and Prescott, E.C., 1982, "Time to Build and Aggregate Fluctuations", *Econometrica,* Vol. 50, pp. 1345-1370

[26] Lucas, Robert E., Jr., 1972, "Expectations and the Neutrality of Money", *Journal of Economic Theory,* Vol. 4, pp. 103-123

[27] Lucas, Robert E., Jr., 1975, "An Equilibrium Model of the Business Cycle", *Journal of Political Economy,* Vol. 83, pp. 1113-1144

[28] Lucas, Robert E., Jr., 1984, "Money in a Theory of Finance", *Carnegie-Rochester Conference Series on Public Policy,* Vol. 21, pp. 9-46

[29] Lucas, Robert E., Jr., 1988, "On the Mechanics of Development Planning", *Journal of Monetary Economics,* Vol. 22, pp. 3-42

[30] Lucas, Robert E., Jr. and Stokey, Nancy L., 1983, "Optimal Fiscal and Monetary Policy in an Economy Without Capital", *Journal of Monetary Economics,* Vol. 12, pp. 55-93

[31] Lucas, Robert E., Jr. and Stokey, Nancy L., 1987, "Money and Interest in a Cash-in-Advance Economy", *Econometrica,* Vol. 55, No. 9, pp. 491-513

[32] McCallum, Bennett T., 1989, "Real Business Cycle Models" in Modern Business Cycle Theory Ed. Robert J. Barro, Harvard University Press, Cambridge, Mass.

[33] McGrattan, Ellen R., 1998, "A Defence of AK Growth Models", Federal Reserve Bank of Minneapolis, Quarterly Review, Fall, pp. 13-26

[34] Orphanides, Athenasios and Sollow, Robert M., 1990, "Money, Inflation, and Growth", in Friedman, Benjamin M. and Hahn, Frank H., Eds., Handbook of Monetary Economics, Amsterdam, North Holland

[35] Poole, Wiliam, 1970, "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model", *Quarterly Journal of Economics,* Vol. 84, pp. 197-216

[36] Rebelo, Sergio, 1991, "Long Run Policy Analysis and Long Run Growth", *Journal of Political Economy,* Vol. 99, No. 3, pp. 500-521

[37] Rodríguez-Mendizábal, Hugo, 1998, "The Variability of Money Velocity in a Generalized Cash-in-Advance Model", Universitat Pompeu Fabra, Working paper 320

[38] Romer, Paul M., 1986, "Increasing Returns and Long Run Growth", *Journal of Political Economy,* Vol. 94, No. 5, part II, pp. S71-S102

[39] Santos, Manuel, 1998, "Numerical Solution of Dynamic Economic Models", Discussion Paper 9804, Centro de Investigación Económica ITAM

[40] Sargent, Thomas J., 1987, Dynamic Macroeconomic Theory, Harvard University Press

[41] Sidrauski, Miguel, 1967, "Rational Choice and Patterns of Growth in a Monetary Economy", *American Economic Review, (Papers and Proceedings),* Vol. 57, No. 2, pp. 534-544

[42] Stockman, Alan C., 1981, "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy", *Journal of Monetary Economics,* Vol. 8, pp. 387-393

[43] Stokey, Nancy L. and Lucas, Robert E., Jr. with Prescott, Edward C., 1989, Recursive Methods in Economic Dynamics, Harvard University Press

[44] Svensson, Lars E. O., 1985, "Money and Asset Prices in a Cash-in-Advance Economy", *Journal of Political Economy,* Vol. 93, No. 5, pp. 919-944

[45] Taylor, John B. and Uhlig, Harald, 1990, "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution methods", *Journal of Business and Economic Statistics,* Vol. 8, No. 1, pp. 1-17

[46] Tobin, James, 1965, "Money and Economic Growth", *Econometrica,* Vol. 33, No. 4, pp. 671-84

[47] Uhlig, Harald, 1997, "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily", CentER, University of Tilburg and CEPR, mimeo

[48] Wang, Ping and Yip, Chong K. "Alternative Approaches to Money and Growth", *Journal of Money, Credit and Banking,* 1992, Vol. 24, No. 4, pp. 553-562

Chapter 2

Variability of the Income Velocity of Money in a Cash-in-Advance Economy with Capital[1]

2.1 Introduction

In this chapter we study the changes in the income velocity of money that arise due to the introduction of a precautionary demand for money. We perform both qualitative and quantitative analysis in a model with productive capital.

There are several effects that may contribute to the variability of the income velocity of money. One is the reaction of real balances to the changes in the nominal interest rates.[2] When the nominal interest rate increases, it is less attractive to hold money and agents look for some alternative means of payment. This fact is captured in the literature by allowing for 'cash' and 'credit' goods, as for example in Lucas (1984), Lucas and Stokey (1983, 1987) and Jones and Manuelli (1995).[3] In such cash-credit models agents have a possibility of substituting between two ways of acquiring consumption goods. Therefore, when the opportunity cost of holding money increases, agents can switch from cash consumption towards credit consumption. This affects the

[1] A part of this chapter appeared as Hromcová, Jana, 1998, "A Note on Income Velocity of Money in a Cash-in-Advance Economy with Capital", *Economics Letters*, Vol. 60, pp. 91-96

[2] Plotting velocity and interest rates, as done for example in McGrattan (1998a), it can be seen that these two variables exhibit a similar pattern.

[3] In the words of Lucas (1994), 'cash' goods are consumed using money and 'credit' goods are exchanged directly for agents savings or are acquired issuing private securities.

real balances, and the income velocity varies accordingly.

Other types of models introduce changes into the money demand exogenously, via a modified cash-in-advance constraint, see Canzoneri and Dellas (1998), Collard, Dellas and Ertz (1998). In their models agents are allowed to exchange a fraction of the current period income for consumption without using money. When this fraction increases, individuals economize on their real balances holdings and this implies a change in the income velocity.

As Lucas (1988) shows, standard cash-in-advance models can explain systematic changes in velocity due to income and interest rates, but not due to other factors. Trends in velocity are attributed to financial innovations and technological improvements in the financial sector and to the fact that the monetary policy influences the decisions of agents to devote resources to the creation of money substitutes (see for example Cole and Ohanian (1997) or Gordon, Leeper and Zha (1998) for surveys on papers which deal with these topics).

Another approach to model variable income velocity can be found in Svensson (1985). In his model agents can consume only in exchange for money, i.e., only 'cash' goods are present. Letting the goods market open before the financial exchange takes place, agents are forced to make their decisions on the money holdings before the state of the economy is known. Such information structure may lead to a precautionary money demand in some states of the world, since agents may end up holding more money than they would need ex-post. Therefore, uncertainty might be a source of fluctuations of the income velocity. Hodrick, Kocherlakota and Lucas (1991) evaluate the stochastic properties of endogenous variables in models which do not necessarily restrict the cash-in-advance constraint to be binding in all states. They analyze cash-only and cash-credit models without capital. They find that in the cash-only model velocity exhibits small variability. Velocity vary in the cash-credit model because agents substitute between cash and credit goods, and the cash-in-advance constraint for the cash good almost always binds.

This chapter extends the analysis of income velocity of real balances done in Svensson (1985) to a model with capital. It will be shown that the properties that hold in a model without capital are also valid in a more general framework. We concentrate on the variability of the income velocity that arises only due to a precautionary money demand. Therefore, we want to use a setup with the timing of events as in Svensson (1985) and disregard other effects that may influence changes in the income velocity. In the model analyzed here, velocity is no longer restricted to values less or equal to unity (as it happens in the model of Svensson (1985)) because part of the output is saved in the form of capital, and money is not needed in order to buy capital.

Clearly, income velocity varies depending on the state of the world. When technology shocks are present, individuals face an uncertainty that makes them demand more real balances than they actually need in order to purchase consumption goods. In this way their cash-in-advance constraint may become nonbinding and the discrepancy between the growth rate of output and that of real balances gives rise to fluctuations in the money velocity.

To perform the analysis we allow for endogenous growth accruing from a linear (AK) technology. Depending on the growth rate of money supply and the technology shock realization, agents might make a precautionary money demand in some periods. We conduct two kinds of analyses. First, we assume identically independently distributed shocks and solve the model analytically. Doing this, we can get some insight on the behavior of the income velocity. For example, we can show how the velocity changes depending on the current state of the world and we derive the highest value for the money velocity. However, we are unable neither to characterize analytically under which conditions a precautionary money demand occurs, nor to evaluate how much variability is introduced by a precautionary money demand into the income velocity of money. Second, we assume that technology shocks are correlated. When correlated shocks are considered, the model has to be solved numerically. We apply the method of parameterized expectations described in Den Haan and Marcet (1990). The reason why we choose this technique is that the algorithm allows to work with nonbinding constraints without any additional difficulty. We see that capital acts like the credit good in Hodrick, Kocherlakota and Lucas (1991), and the model can deliver reasonable velocity fluctuations when the cash-in-advance constraint binds and when technology shocks are correlated. Nevertheless, our main concern is to evaluate the changes in the income velocity when the cash-in-advance constraint is not always binding. We find that a precautionary money demand does not introduce a significant decrease into the income velocity.

The remainder of the chapter is organized as follows. The model is described in section 2.2. In section 2.3 we solve the model analytically and state some conclusions about velocity behavior. In section 2.4 we calibrate the model to the US economy and solve it numerically. We analyze the equilibrium behavior of the economy. In particular, we study the changes in the income velocity caused by a precautionary money demand. Final conclusions are summarized in section 2.5.

2.2 Model Description

2.2.1 The Household Problem

We consider a representative agent economy. There is one consumption good which must be paid using currency. The economy has an infinitely lived representative household with preferences represented by the utility function

$$E_t\left\{\sum_{j=t}^{\infty}\beta^{j-t}\left(\frac{c_j^{1-\theta}-1}{1-\theta}\right)\right\} \tag{2.1}$$

where c_j is the consumption in period j and $\theta > 0$ can be viewed either as the inverse of the elasticity of intertemporal substitution or as the index of relative risk aversion.

The production and trade is characterized like in Lucas and Stokey (1983). Each household is composed of a worker-shopper pair. The timing of the events is the following. Agents enter the period t with a certain amount of monetary balances M_t and bonds B_t, carried over from the previous period, and the capital stock k_t, which is the result of the previous period saving. A representative worker produces via the net production function

$$y_t = A_t k_t \tag{2.2}$$

where A_t is the shock to technology.[4] Prior to any trading agents learn the state of the economy A_t. The production takes place and y_t is realized. The worker stays at the place of production during the whole period. Goods market opens and consumption takes place. The shopper visits various stores to acquire consumption goods carrying the monetary balances of the household. Purchases are subject to the liquidity constraint

$$c_t \leq \frac{M_t}{p_t}. \tag{2.3}$$

The part of the production which is not consumed is saved and used as the next period capital k_{t+1}. Next, the monetary transfer $X_t = (\mu - 1)\,M_t$ from the government takes place. We denote μ as the growth rate of money supply which is set constant for all

[4]To justify the constant return assumption we typically interpret the capital stock as a broad measure that may include also human capital. The class of models that reduce to an AK model are discussed in chapter 4 of Barro and Sala-i-Martin (1995). See also McGrattan (1998b) for the defence of AK models in growth theory.

time periods. At the end of the period asset market opens. Agents can purchase one-period pure discount bonds paying B_{t+1} units of money at period t that pay $R_{t+1}B_{t+1}$ units of money when they mature in $t+1$. Therefore, R_{t+1} is the gross nominal interest rate between period t and $t+1$. Bonds are in zero net supply. Monetary balances M_{t+1} to be carried over to the next period are chosen. The budget constraint that agents are facing is thus

$$c_t + k_{t+1} + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{p_t} \leq A_t k_t + \frac{M_t}{p_t} + \frac{R_t B_t}{p_t} + \frac{X_t}{p_t}. \tag{2.4}$$

The real wealth of a household in period t is in the right hand side of (2.4) and consists of the current period output, real monetary balances, real return on bonds and the real lump sum transfer from the government. Current period wealth is used to consume, and to save in the form of capital, money and bonds as can be seen in the left hand side of (2.4).

A representative household is maximizing the expected discounted utility (2.1) subject to the budget constraint (2.4) and the cash-in-advance constraint (2.3). The stochastic sequences $\{c_t, M_{t+1}, k_{t+1}, B_{t+1}\}_{t=1}^{\infty}$ are to be chosen given the initial monetary balances M_1, the initial capital k_1, and the initial bonds B_1.

2.2.2 Equilibrium

Let λ_t and η_t be the non-negative Lagrange multipliers associated with the budget constraint (2.4) and the cash-in-advance constraints (2.3), respectively. The equations that characterize the equilibrium are the first order conditions

$$\begin{aligned}
c_t^{-\theta} &= \lambda_t + \eta_t, & (2.5)\\
\frac{\lambda_t}{p_t} &= \beta E_t\left(\frac{\lambda_{t+1} + \eta_{t+1}}{p_{t+1}}\right), & (2.6)\\
\lambda_t &= \beta E_t\left(\lambda_{t+1} A_{t+1}\right), & (2.7)\\
\frac{\lambda_t}{p_t} &= \beta R_{t+1} E_t\left(\frac{\lambda_{t+1}}{p_{t+1}}\right) & (2.8)
\end{aligned}$$

and the following transversality conditions:

$$\begin{aligned}
\lim_{j\to\infty} E_t\left(\beta^{t+j}\lambda_{t+j}k_{t+j+1}\right) &= 0, & (2.9)\\
\lim_{j\to\infty} E_t\left(\beta^{t+j}\lambda_{t+j}\frac{B_{t+j+1}}{p_{t+j}}\right) &= 0. & (2.10)
\end{aligned}$$

The Lagrange multipliers associated with the budget and cash-in-advance constraints, λ_t and η_t can be interpreted as the marginal utility of wealth and the marginal utility of real balances, respectively. The first order condition on consumption (2.5) indicates that the existence of binding liquidity constraint drives a wedge between the marginal utility of wealth and the marginal utility of consumption since the wealth cannot be used instantaneously to buy consumption. The first order condition on nominal balances (2.6) is the equation for pricing money. Money is priced like other assets, once its return has been appropriately defined in terms of the liquidity services it provides. The first order conditions on capital (2.7) and bonds (2.8) embody the costs and profits associated with investing one marginal unit of wealth in capital and bonds, respectively.

Definition: Given the set of initial conditions M_1, k_1, B_1 and the growth rate of the money supply μ the equilibrium consists of stochastic processes $\{c_t, M_{t+1}, k_{t+1}, B_{t+1}, R_{t+1}, p_t\}_{t=1}^{\infty}$ such that

(a) a representative household chooses the stochastic sequences $\{c_t, M_{t+1}, k_{t+1}, B_{t+1}\}_{t=1}^{\infty}$ in order to maximize the expected discounted utility (2.1) subject to the budget constraint (2.4) and the cash-in-advance constraint (2.3),

(b) markets for goods, money and bonds clear in every period,

$$c_t + k_{t+1} = A_t k_t, \tag{2.11}$$

$$M_{t+1} = \mu M_t, \tag{2.12}$$

$$B_{t+1} = 0. \tag{2.13}$$

2.2.3 Model Transformation

Since we deal here with an endogenous growth model, variables are not stationary. Therefore, we will work instead with variables in ratios. We define $\hat{c}_t$ as the ratio of consumption and current period capital and $\hat{k}_{t+1}$ as the gross rate of growth of capital,

$$\hat{c}_t = \frac{c_t}{k_t} \text{ and } \hat{k}_{t+1} = \frac{k_{t+1}}{k_t}.$$

Then, we define $\hat{m}_t$ as the ratio of real monetary balances and capital

$$\hat{m}_t = \frac{m_t}{k_t}$$

where the real monetary balances are defined as the ratio of nominal balances and prices

$$m_t = \frac{M_t}{p_t}. \tag{2.14}$$

Stationary Lagrange multipliers $\hat{\lambda}_t$ and $\hat{\eta}_t$ can be written as

$$\hat{\lambda}_t = \lambda_t k_t^{\theta} \text{ and } \hat{\eta}_t = \eta_t k_t^{\theta}.$$

We can write the system of equations that characterize the equilibrium, that is, the first order conditions (2.5)-(2.8), the cash-in-advance constraint (2.3) and the equilibria in goods, money and bonds markets (2.11)-(2.13), in terms of the new variables in the following form:

$$\begin{aligned}
\hat{c}_t^{-\theta} &= \hat{\lambda}_t + \hat{\eta}_t, && (2.15)\\
\hat{\lambda}_t \hat{m}_t &= \frac{\beta}{\mu} E_t \left[\left(\hat{\lambda}_{t+1} + \hat{\eta}_{t+1} \right) \hat{m}_{t+1} \hat{k}_{t+1}^{1-\theta} \right], && (2.16)\\
\hat{\lambda}_t &= \beta E_t \left(\hat{\lambda}_{t+1} A_{t+1} \hat{k}_{t+1}^{-\theta} \right), && (2.17)\\
\frac{\hat{\lambda}_t \hat{m}_t}{R_{t+1}} &= \frac{\beta}{\mu} E_t \left[\hat{\lambda}_{t+1} \hat{m}_{t+1} \hat{k}_{t+1}^{1-\theta} \right], && (2.18)
\end{aligned}$$

$$\begin{aligned}
(\hat{c}_t - \hat{m}_t)\hat{\eta}_t &= 0, \quad \text{and} \quad \hat{\eta}_t \geq 0, && (2.19)\\
\hat{c}_t + \hat{k}_{t+1} &= A_t, && (2.20)\\
M_{t+1} &= \mu M_t, && (2.21)\\
B_{t+1} &= 0 && (2.22)
\end{aligned}$$

where (2.16) and (2.18) are obtained by plugging the money market equilibrium (2.12) and the relationship between real and nominal balances (2.14) into the first order conditions on money (2.16) and bonds (2.18), respectively.

2.3 The Case of Identically Independently Distributed Shocks

We assume first that the technology shocks are lognormally identically independently distributed

$$\ln A_t \sim N(\ln \bar{A}, \sigma_A^2).$$

This assumption allows us to treat the model analytically and get some insights on the velocity behavior. In order to solve the model we will look for the policy functions for capital and consumption by assuming that they have the following functional forms

$$\begin{aligned} \hat{k}_{t+1} &= ZA_t, && (2.23) \\ \hat{c}_t &= (1-Z)A_t. && (2.24) \end{aligned}$$

Plugging (2.23) and (2.24) into the first order conditions we get that

$$Z = \beta^{\frac{1}{\theta}} E_t \left(A_{t+1}^{1-\theta}\right)^{\frac{1}{\theta}}$$

where $E_t\left(A_{t+1}^{1-\theta}\right)$ is constant under the assumption of identically independently distributed technology shocks. Because there are no monetary disturbances, investment and consumption are independent of the money growth rate. If the cash-in-advance constraint is binding, then $\hat{c}_t = \hat{m}_t$, and

$$\hat{m}_t = (1-Z)A_t. \tag{2.25}$$

When agents do not spend all monetary balances and make a precautionary money demand, the cash-in-advance constraint does not bind. In that case $\hat{c}_t < \hat{m}_t$ and $\hat{\eta}_t = 0$ and we can express the real money demand using the first order condition on nominal balances (2.16) as

$$\hat{m}_t = \frac{\beta}{\mu} E_t \left[\frac{\left(\hat{\lambda}_{t+1} + \hat{\eta}_{t+1}\right) \hat{m}_{t+1} \hat{k}_{t+1}^{1-\theta}}{\hat{\lambda}_t} \right]. \tag{2.26}$$

The optimal consumption plan is given by the expression (2.24). When the cash-in-advance constraint becomes nonbinding, and (2.26) applies, agents do not spend all balances accumulated for consumption. Therefore, the precautionary money demand in (2.26) must be higher than the money demand in (2.25). However, if one wants to solve explicitly for the precautionary money demand, a numerical solution has to be applied.

Income velocity of money v_t is the ratio between real output (2.2) and real monetary balances (2.14). In terms of the transformed model we can write

$$v_t = \frac{A_t}{\hat{m}_t}. \tag{2.27}$$

We know that when the cash-in-advance constraint becomes nonbinding, the real money demand is higher than when the cash-in-advance constraint binds. This implies a lower income velocity in those periods in which agents make a precautionary money demand. Therefore, taking into account (2.25) we can write

$$v_t \leq \frac{1}{1-Z} \tag{2.28}$$

where either the equality or the inequality in equation (2.28) holds depending on the cash-in-advance constraint being binding or nonbinding, respectively. The value $1/(1-Z)$ is the highest value for the income velocity and such a value is reached in all states in which the cash-in-advance constraint binds. Notice that the velocity does not vary when the cash-in-advance constraint always binds, because Z is a constant. Velocity always varies due to shocks in technology when the cash-in-advance constraint becomes nonbinding.

2.4 The Case of Correlated Shocks

2.4.1 Solution Method

We assume now that the logarithm of technology shocks follows an autoregressive process,

$$\ln A_t = (1-\rho_A)\ln \bar{A} + \rho_A \ln A_t + \varepsilon_{A,t}, \tag{2.29}$$

where $\bar{A} > 0$ is the steady state value of the technology, $0 < \rho_A < 1$ and $\varepsilon_{A,t}$ is a white noise, $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$. To solve the model we need to solve the system of equilibrium equations which consists of the first order conditions (2.15)-(2.18), the goods, money and bonds market equilibrium equations (2.20)-(2.22), and the cash-in-advance constraint (2.19). We are interested in equilibria in which a precautionary money demand occurs in some periods. That means that we will work with equilibria where the cash-in-advance constraint becomes nonbinding in some states of the world. The method of parameterized expectations described in detail in Den Haan and Marcet (1990) will be applied. This technique is useful in our case as there is no additional difficulty when working with nonbinding constraints. An application of this method to a monetary model can be found for example in Den Haan (1990) or Rodríguez-Mendizábal (1998).

Our task is now to solve the system (2.15)-(2.22) applying the numerical algorithm mentioned above. The method consists of approximating expectations in equilibrium equations by flexible functional forms. In our model there are three expectations to be parameterized. They are the ones appearing in the equations (2.16), (2.17) and (2.18). A functional form that approximates an expectation should be a function of state variables known at time t entering in the agents' information set. In the original model we had one exogenous state variable A_t and one endogenous state variable k_t. However, after transforming the model, $\hat{k}_t$ is only dependent on the technology shocks and therefore, it has no additional predictive power. The functional form that approximates the expectations (2.16)-(2.18) will be only a function of the technology shock A_t. We will approximate the expectations by second degree exponenciated polynomials

$$\begin{aligned} E_t\left[\left(\hat{\lambda}_{t+1}+\hat{\eta}_{t+1}\right)\hat{m}_{t+1}\hat{k}_{t+1}^{1-\theta}\right] &= \psi_1\left(A_t;a\right)=\exp\left[a_1+a_2\ln A_t+a_3\ln A_t^2\right] &(2.30)\\ E_t\left(\hat{\lambda}_{t+1}A_{t+1}\hat{k}_{t+1}^{-\theta}\right) &= \psi_2\left(A_t;b\right)=\exp\left[b_1+b_2\ln A_t+b_3\ln A_t^2\right], &(2.31)\\ E_t\left(\hat{\lambda}_{t+1}\hat{m}_{t+1}\hat{k}_{t+1}^{1-\theta}\right) &= \psi_3\left(A_t;d\right)=\exp\left[d_1+d_2\ln A_t+d_3\ln A_t^2\right] &(2.32)\end{aligned}$$

where $a=(a_1,a_2,a_3)'$, $b=(b_1,b_2,b_3)'$ and $d=(d_1,d_2,d_3)'$ are the vectors of parameters.

To apply the algorithm we first have to simulate time series for the exogenous process $\{A_t\}_{t=1}^T$, for large T. For some initial values of the parameter vectors a, b and d, say a^0, b^0 and d^0, the model can be solved and time series for the endogenous process can be obtained.[5] For any given values of a^0, b^0 and d^0 new values a^1, b^1 and d^1 can be found by running nonlinear least squares regressions on

$$\left(\hat{\lambda}_{t+1}+\hat{\eta}_{t+1}\right)\hat{m}_{t+1}\hat{k}_{t+1}^{1-\theta},$$

$$\hat{\lambda}_{t+1}A_{t+1}\hat{k}_{t+1}^{-\theta},$$

and

$$\hat{\lambda}_{t+1}\hat{m}_{t+1}\hat{k}_{t+1}^{1-\theta}.$$

The standard implementation of the parameterized expectations algorithm finds the best approximation by iterating on this process. A more detailed description of the application of the algorithm can be found in the Appendix of this chapter.

[5]Initial values are set using the homotopy method as suggested in Marcet (1991). We begin with parameters obtained from the analytical solution which is available for the logarithmic utility function and for binding cash-in-advance constraint, i.e. $\theta=1$ and for a high enough value of μ.

2.4.2 Calibration

We can write the system of equilibrium equations of the model (2.15)-(2.20) in the non-stochastic steady state as follows:

$$\begin{aligned}
\bar{c}^{-\theta} &= \bar{\lambda} + \bar{\eta}, && (2.33)\\
\bar{\lambda} &= \frac{\beta}{\mu}\left(\bar{\lambda} + \bar{\eta}\right)\bar{k}^{1-\theta}, && (2.34)\\
1 &= \beta \bar{A}\bar{k}^{-\theta}, && (2.35)\\
\mu &= \beta \bar{R}\bar{k}^{1-\theta}, && (2.36)\\
\bar{c} &= \bar{m}, && (2.37)\\
\bar{c} + \bar{k} &= \bar{A} && (2.38)
\end{aligned}$$

where the variables with a *bar* denote their steady state values.[6] We calibrate the model to match the quarterly US data following Collard, Dellas and Ertz (1998). The average growth rate is 0.7%, i.e., $\bar{k} = 1.007$. We will set $\beta = 0.99$. All steady state values can be determined from the system (2.33)-(2.38), for any value of the parameter θ. We will consider that the relative risk aversion parameter takes the values in the interval $\theta \in [0.7, 2]$. The correlation coefficient ρ_A is chosen to be 0.95 and the standard error of technology shocks is set to $\sigma_A = 0.007$ like in Cooley and Prescott (1995). The average growth rate of money (M1) in the data is $\bar{\mu} = 1.008$.

2.4.3 Precautionary Money Demand

Money demand is given by equations (2.16) and (2.19) and it is a function of the money growth rate, the discounting rate and the current technology shock.[7] When the money growth rate increases, the steady state value of the nominal interest rate also increases, as can be seen in equation (2.36). This means that it is less attractive to hold money and it is more attractive to spend the monetary balances on consumption. Increasing the money growth rate will result in a binding cash-in-advance constraint in more states of the world. On the other hand, decreasing the money growth rate will allow us to find nonbinding liquidity constraints in more periods.

[6] Non-stochastic steady state of the transformed model, i.e., when $\sigma_A = 0$, represents the balanced growth path of the original model.

[7] The growth rate of capital $\hat{k}_{t+1}$, the transformed marginal utility of wealth $\hat{\lambda}_t$ and the process of expectation formation are all functions of the realization of the technology shock in period t.

According to our simulations, precautionary money demand arises under different conditions for $\theta < 1$ and for $\theta > 1$. When the parameter θ is lower than unity, agents tend to make a precautionary money demand when a technology shock is high. The opposite holds when θ is higher than unity, that is, for a sufficiently high shock the cash-in-advance constraint tends to become binding.

The reaction of the money demand and the nominal interest rate to the technology shocks (for a given money growth rate) is plotted in Figure 2.1.[8] In any period t for which the ratio $c_t/m_t < 1$, agents make a precautionary money demand. The first row of Figure 2.1 shows the evolution of the technology shocks, the second row shows the evolution of c_t/m_t in comparison to the nominal interest rate R_{t+1}. We have performed the simulations for values of $\theta \in [0.7, 2]$. In the left and right columns of Figure 2.1 the behavior for $\theta = 0.7$ and for $\theta = 1.5$ is plotted. This two values illustrate the behavior for $\theta < 1$ and $\theta > 1$, respectively.

Agents with high θ want to smooth their consumption path. When there is a positive technology shock, a direct way of isolating the effect of the shock on consumption is to make the cash-in-advance constraint binding, so that consumption is prevented from reacting too much to output fluctuations. When the money growth rate μ is sufficiently high, individuals never make a precautionary money demand because the opportunity cost of holding money is high, as argued in the first paragraph of this section. When $\theta > 1$, we find that the technology shock A_t is positively correlated with the nominal interest rate R_{t+1} (see the right column of Figure 2.1). This implies that the opportunity cost of holding money decreases in bad times. For low money growth rates agents might make a precautionary money demand when A_t is low. All unspent balances are the part of the current period wealth. They cannot be saved in the form of capital immediately because at the time when the t period financial exchange takes place, the decision on k_{t+1} has been already taken. But the balances unspent at time t can be saved as k_{t+2}. This will accelerate the capital accumulation which will imply an increase in output and consumption. In this way the fluctuations in consumption can be dampened.

When $\theta < 1$, agents care less about consumption smoothing. Because the nominal interest rate R_{t+1} is negatively correlated with technology shock A_t (see the left column of Figure 2.1), it is more attractive to hold money in good times. Therefore the cash-in-advance constraint may become nonbinding when A_t is high.

[8] The nominal interest rate is a function of the money growth rate, the discounting rate and the technology shocks, as can be seen in equation (2.18).

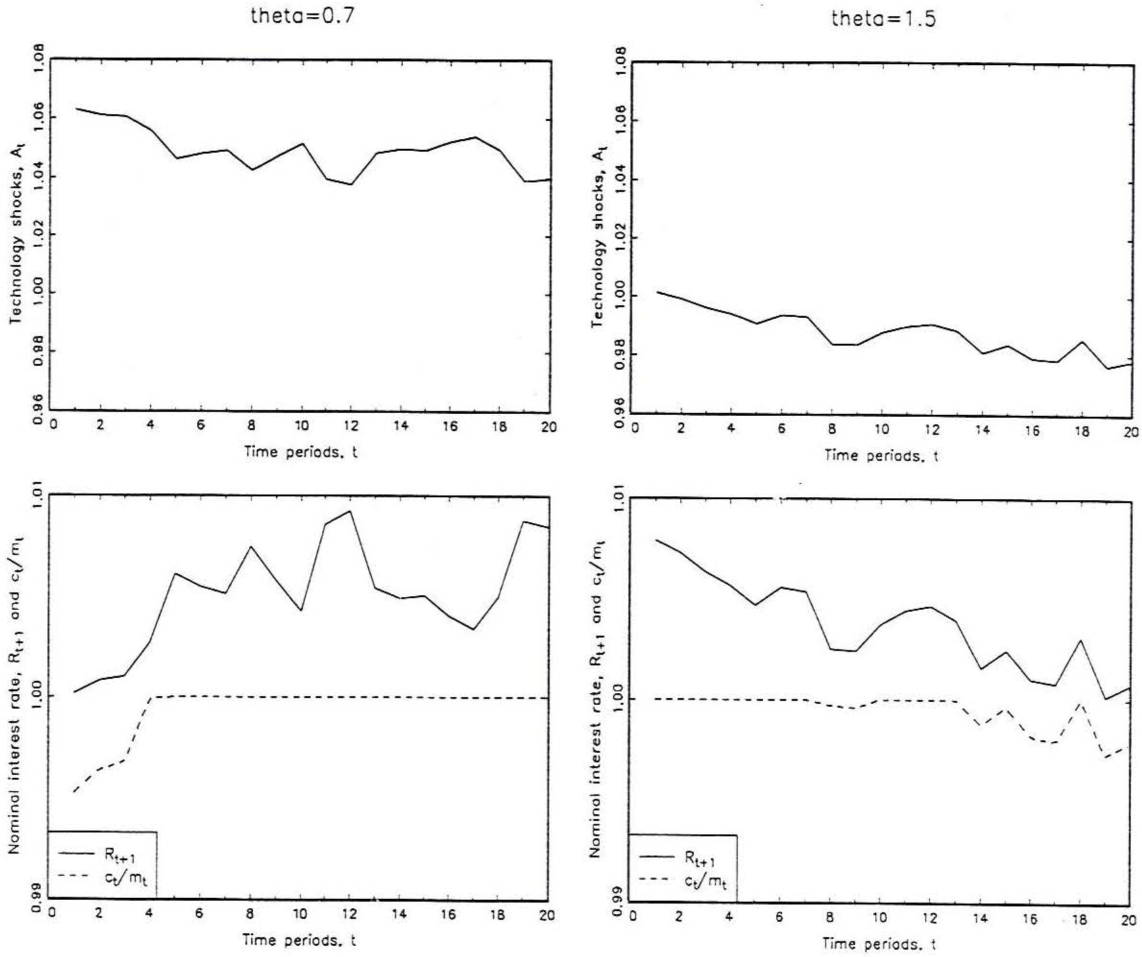

Figure 2.1: Relationship between technology shocks A_t, precautionary money demand (consumption to real balances ratio c_t/m_t), and the nominal interest rate R_{t+1}, for $\beta = 0.99$, for the relative risk aversions lower and higher than unity and a given money growth rate:

a) Left column: results for $\theta = 0.7$, $\mu = 1.01$,

b) Right column: results for $\theta = 1.5$, $\mu = 1.00$.

2.4.4 Velocity Analysis

We want to evaluate the changes in the income velocity and its volatility due to the fact that the agents make a precautionary money demand. Our benchmark case will be an economy with the lowest money growth rate in which the cash-in-advance constraint binds in all periods, i.e., agents never make any precautionary money demand. We then compare the benchmark case to the ones in which the cash-in-advance constraint does not bind in some periods.

The income velocity of money is defined as in (2.27). The volatility of the income velocity of money will be calculated as a standard error of the velocity prediction

$$\sigma_v = \left\{ E_{t-1} \left(\left[\ln v_t - E_{t-1} \left(\ln v_t \right) \right]^2 \right) \right\}^{\frac{1}{2}} \tag{2.39}$$

which corresponds to calculating the standard error of the residuals of the regression

$$\ln v_t = \zeta_v + \rho_v \ln v_{t-1} + \varepsilon_{v,t}. \tag{2.40}$$

We denote σ_v^{sim} and σ_v^{data} as the standard errors found in simulated series and in the data, respectively.

Writing the expression for the income velocity for the non-stochastic steady state

$$\bar{v} = \frac{\bar{A}}{\bar{m}}$$

and solving for $\bar{m}$ from the system (2.33)-(2.38) for $\bar{\eta} > 0$ we can see that the average value of money velocity is independent of the money growth rate. By changing the money growth rate μ we may generate stochastic equilibria in which the cash-in-advance constraint always binds (and the average velocity is independent of μ) or becomes nonbinding in some periods (and the average velocity may depend on μ). As it has been already described, higher values of μ will lead to higher average values of nominal interest rates $\bar{R}$. Therefore, the actual nominal interest rates R_{t+1} will oscillate around this higher value of $\bar{R}$ and the opportunity cost of holding money will be higher, so that we can expect that a precautionary money demand will appear with less probability. Lowering μ will lead to a precautionary money demand in more states of the world. Applying the same argument as the one in section 2.3, a precautionary money demand implies a lower income velocity. Therefore, we will lower the growth rate of money supply in order to observe which changes are introduced into the behavior of the income velocity when the cash-in-advance constraint becomes nonbinding more frequently.

θ	μ	$NONB^*$ [%]	$\nabla\bar{v}^*$ [%]	σ_v^{sim*}
0.7	1.03	0	-	0.0500
0.7	1.01	4	0.53	0.0497
0.7	0.997	68	6.3	0.033
0.7	0.993	99	35.62	0.0058
0.9	1.00	0	-	0.01266
0.9	0.999	0.014	0.065	0.01265
0.9	0.992	68	1.30	0.0088
0.9	0.991	94.2	4.71	0.0042
1.1	1.0	0	-	0.01027
1.1	0.996	0.03	0.026	0.01025
1.1	0.9905	70.4	1.18	0.0067
1.1	0.99	86	2.5	0.0047
1.2	1.01	0	-	0.0188
1.2	1.0	0.07	0.51	0.0188
1.2	0.992	51	1	0.0158
1.2	0.99	86	4.6	0.0086
1.5	1.02	0	-	0.03766
1.5	1.01	0.08	0.33	0.03767
1.5	1	5.6	0.09	0.03764
1.5	0.991	75.5	5.9	0.0233
1.5	0.99	86.7	9.25	0.0177
2.0	1.02	0.08	-	0.0586
2.0	1.01	0.3	0.25	0.0585
2.0	0.993	60	4.9	0.0475
2.0	0.99	89	15	0.0273

Table 2.1: Changes in the income velocity and its volatility due to a precautionary money demand: θ – relative risk aversion, μ – money growth rate, $NONB$ – % of periods in which a precautionary money demand arises, $\nabla\bar{v}$ – % decrease in the income velocity of money compared to the benchmark case, σ_v^{sim} – volatility of the income velocity obtained from simulated series, * – denotes averages over 500 shock realizations

In Table 2.1 we report the following variables for different values of θ and different money growth rates μ: percentage of periods in which a precautionary money demand occurs, *NONB*, percentage decrease in the average income velocity of money compared to the benchmark case, $\nabla \bar{v}$, and the volatility of the income velocity obtained from simulated series, σ_v^{sim}.[9] We observe that the decrease in the average income velocity caused by a precautionary money demand is rather low. Velocity drops for about 2-10% when the cash-in-advance constraint becomes nonbinding in most of the states, more than 50% of periods.

Comparing the values of the simulated volatilities, to the value we find in the US data, $\sigma_v^{data} = 0.0178$, (we calculate the velocity as a ratio of nominal GDP and M1 monetary aggregates) we may say that the corresponding relative risk aversion parameter θ should be set to a value around 1.2.[10] Taking into account the average money growth rate found in data, $\bar{\mu} = 1.008$, we see that it does not seem very likely that the individuals make a precautionary money demand in many periods.

2.5 Conclusion

We have used a simple stochastic growth model with money introduced via a cash-in-advance constraint to analyze the behavior of the income velocity of real balances. The analysis of the precautionary money demand shows that the result of Svensson (1985) holds in a more general framework. Compared to Svensson (1985), however, the presence of capital in the model reverses the relationship between the precautionary money demand and technology shocks. In the model of Svensson (1985) agents who want to smooth their consumption path might make a precautionary money demand in good times. By spending the extra balances in bad times they avoid high consumption fluctuations.[11] In the model with sustained growth analyzed here, agents keep the cash-in-advance constraint binding under a positive technology shock and thus prevent

[9] Velocity is stationary, because it is a function of stationary variables A_t and $\hat{m}_t$. We calculate the average velocity for the benchmark case, which is the one when the cash-in-advance constraint binds in all periods. The decrease in the velocity caused by a precautionary money demand is calculated comparing the average velocity under the benchmark case to the average velocity in the analyzed one.

[10] Estimates for θ from various studies rather vary. For example, Canzoneri and Dellas (1998) propose a reasonable range of θ between 1 and 4.

[11] Svensson (1985) develops the result assuming identically independently distributed technology shocks. Nevertheless, performing a numerical analysis we can show that the result remains unchanged for correlated technology shocks.

that consumption reacts too much to output fluctuations. Individuals might make a precautionary money demand in bad times. Unspent balances can be saved in the future period in the form of capital. Higher capital accumulation accelerates growth and the effects of low shocks on consumption can be thus slightly dampened.

Concerning the volatility of the income velocity, the model can deliver empirically plausible velocity fluctuations when the cash-in-advance constraint is binding. That implies that in our economy capital acts like credit good in Hodrick, Kocherlakota and Lucas (1991). We find that in more practical terms the presence of a precautionary money demand does not lead to quantitatively important changes in the income velocity of money.

2.6 Appendix

The parameterization consists of approximating the expectations in the equilibrium equations (2.16), 2.17) and (2.18) by functions $\psi_1(A_t;a)$, $\psi_2(A_t;b)$ and $\psi_3(A_t;d)$, where a, b and d are the vectors of parameters. The crucial part of the solution procedure is to find functions $\psi_1(A_t;a)$, $\psi_2(A_t;b)$ and $\psi_3(A_t;d)$, and vectors a, b and d. Polynomial functions work rather well approximating any arbitrary functions. We use the exponenciated polynomials, given in (2.30)-(2.32) to assure that the expectations we parameterize are positive.

Once the expectations are parameterized, the system of equilibrium equations (2.15)-(2.22) becomes dependent on the parameters a, b and d. We rewrite such equilibrium equations in a convenient form. The algorithm solves for the endogenous processes as follows:

$$\begin{aligned}
\hat{\lambda}_t(a,b,d)\ \hat{m}_t(a,b,d) &= \frac{\beta}{\mu}\psi_1(A_t;a), && (2.41)\\
\hat{\lambda}_t(a,b,d) &= \beta\psi_2(A_t;b), && (2.42)\\
\hat{m}_t(a,b,d) &= \frac{\psi_1(A_t;a)}{\mu\psi_2(A_t;b)}, && (2.43)
\end{aligned}$$

$$\left\{\begin{array}{l}
\text{a) } \hat{c}_t(a,b,d)=\hat{m}_t(a,b,d),\ \hat{\eta}_t(a,b,d)=\hat{c}_t(a,b,d)^{-\theta}-\hat{\lambda}_t(a,b,d) \quad \text{if} \quad \hat{\eta}_t(a,b,d)>0\\
\text{or}\\
\text{b) } \hat{\eta}_t(a,b,d)=0,\ \hat{c}_t(a,b,d)=\hat{\lambda}_t(a,b,d)^{-\frac{1}{\theta}} \quad \text{if} \quad \hat{c}_t(a,b,d)<\hat{m}_t(a,b,d),
\end{array}\right. \tag{2.44}$$

$$\begin{aligned}
R_{t+1}(a,b,d) &= \frac{\psi_1(A_t;a)}{\psi_3(A_t;d)}, && (2.45)\\
\hat{k}_{t+1}(a,b,d) &= A_t-\hat{c}_t(a,b,d), && (2.46)\\
k_{t+1}(a,b,d) &= \hat{k}_{t+1}(a,b,d)k_t(a,b,d), && (2.47)\\
M_{t+1} &= \mu M_t, && (2.48)\\
p_t(a,b,d) &= \frac{M_t}{\hat{m}_t(a,b,d)k_t(a,b,d)} && (2.49)
\end{aligned}$$

To apply the algorithm we have to simulate time series for the exogenous process $\{A_t\}_{t=1}^{T}$, for large T, set the initial values for capital k_1 and money stock M_1 and the growth rate of money supply μ.[12] The system (2.41)-(2.49) can be solved for any given

[12] The number of observations used is $T = 10000$.

a, b and d, and we obtain the sequences $\{\hat{c}_t, \hat{k}_{t+1}, \hat{m}_t, \hat{\lambda}_t, \hat{\eta}_t, k_{t+1}, R_{t+1}, M_{t+1}, p_t\}_{t=1}^{T}$.[13] The second step is to look for solutions of the following minimization problems:

$$S_1(a) = \arg\min_{\bar{a}} E\left\{\left[\left(\hat{\lambda}_{t+1}(a,b,d) + \hat{\eta}_{t+1}(a,b,d)\right)\hat{m}_{t+1}(a,b,d)\hat{k}_{t+1}(a,b,d)^{1-\theta}\right] - \psi_1\left(A_t;\bar{a}\right)\right\},$$

$$S_2(b) = \arg\min_{\bar{b}} E\left\{\left[\hat{\lambda}_{t+1}(a,b,d)A_{t+1}\hat{k}_{t+1}(a,b,d)^{-\theta}\right] - \psi_2\left(A_t;\bar{b}\right)\right\}$$

and

$$S_3(d) = \arg\min_{\bar{d}} E\left\{\left[\hat{\lambda}_{t+1}(a,b,d)\hat{m}_{t+1}(a,b,d)\hat{k}_{t+1}(a,b,d)^{1-\theta}\right] - \psi_3\left(A_t;\bar{d}\right)\right\}.$$

We want to choose the parameters a, b and d such that $S_1(a) = a$, $S_2(b) = b$ and $S_3(d) = d$. Given the series $\{\hat{k}_{t+1}, \hat{m}_t, \hat{\lambda}_t, \hat{\eta}_t, A_t\}_{t=1}^{T}$, the parameters a, b and d are found by minimizing the mean squared error, that is, we find the values g_i for $i = 1, 2, 3$, that minimize

$$\frac{1}{T-1}\sum_{t=1}^{T-1}\{\phi_{it} - \psi_i\left(A_t;g_i\right)\}^2$$

where

$$\phi_{1t} = \left[\hat{\lambda}_{t+1}(a,b,d) + \hat{\eta}_{t+1}(a,b,d)\right]\hat{m}_{t+1}(a,b,d)\hat{k}_{t+1}(a,b,d)^{1-\theta},$$

$$\phi_{2t} = \hat{\lambda}_{t+1}(a,b,d)A_{t+1}\hat{k}_{t+1}(a,b,d)^{-\theta}$$

and

$$\phi_{3t} = \hat{\lambda}_{t+1}(a,b,d)\hat{m}_{t+1}(a,b,d)\hat{k}_{t+1}(a,b,d)^{1-\theta}.$$

For large T, g_i are the approximations to $S_i(l)$ for $i = 1, 2, 3$ and $l = a, b, d$. In practice this amounts to apply nonlinear least squares to estimate the parameters g_i for $i = 1, 2, 3$.

We solve the model for some initial parameter vectors a^0, b^0 and d^0. Using the values from the ith iteration, the values of the parameters for the $(i+1)$th iteration are calculated according to

$$l^{i+1} = (1-\Lambda)l^i + \Lambda S(l^i)$$

[13] To decide which step to follow in equation (2.44) we just have to check which of the two cases holds, a) or b). At the same time we make sure that only one of the cases holds.

where $l = a, b, d$ and $\Lambda \in (0,1]$ is selected to be equal to its maximal feasible value that assures convergence ($\Lambda = 1$ works well in this case). The process of calculating new series from (2.41)-(2.49) is repeated until the functions $\psi_1(A_t; a)$, $\psi_2(A_t; b)$ and $\psi_3(A_t; d)$ approximate the expectations with a required level of precision. In our case the calculated parameters must fulfill

$$\sqrt{\sum_{j=1}^{3}\left(a_j^{i+1} - a_j^i\right)^2 + \sum_{j=1}^{3}\left(b_j^{i+1} - b_j^i\right)^2 + \sum_{j=1}^{3}\left(d_j^{i+1} - d_j^i\right)^2} < 0.0001$$

The original series in levels can be obtained by reversing the transformation presented in section 2.2.3.

Bibliography

[1] Barro, J. Robert and Sala-i-Martin, Xavier, 1995, Economic Growth, McGraw-Hill

[2] Cooley, Thomas F. and Prescott, Edward C., 1995, "Economic Growth and Business Cycles", in Frontier of Business Cycle Research, Ed. Thomas F. Cooley, Princeton University Press

[3] Canzoneri, Matthew B. and Dellas, Harris, 1998, "Real Interest and Central Bank Operating Procedures", *Journal of Monetary Economics,* Vol. 42, pp. 471-494

[4] Collard, Fabrice, Dellas, Harris and Ertz, Guy, 1998, "Poole Revisited", IRES, Université Catolique de Louvain, Discussion Paper 9823

[5] Cole, Harold L. and Ohanian, Lee E., 1997, "Shrinking Money and Monetary Business Cycles", Federal Reserve Bank of Minneapolis, Research Department Working Paper 579

[6] Den Haan, Wouter J., 1990, "The Optimal Inflation Path in a Sidrauski-Type Model with Uncertainty", *Journal of Monetary Economics,* Vol. 25, pp. 389-409

[7] Den Haan, Wouter J. and Marcet, Albert, 1990, "Solving the Stochastic Growth Model by Parameterizing Expectations", *Journal of Business and Economic Statistics,* Vol. 8, No. 1, pp. 31-34

[8] Gordon, David B., Leeper Eric M. and Zha, Tao, 1998, "Trends in Velocity and Policy Expectations", *Carnegie-Rochester Conference Series on Public Policy,* Vol. 49, pp. 265-304

[9] Hodrick, Robert J., Kocherlakota, Narayana and Lucas, Deborah, 1991, "The Variability of Velocity in Cash-in-Advance Models", *Journal of Political Economy,* Vol. 99, No. 2, pp. 258-384

[10] Jones, Larry E. and Manuelli, Rodolfo E., 1995, "Growth and the Effects of Inflation", *Journal of Economic Dynamics and Control,* Vol. 19, pp. 1405-1428

[11] Lucas, Robert E., Jr., 1984, "Money in a Theory of Finance", *Carnegie-Rochester Conference Series on Public Policy,* Vol. 21, pp. 9-46

[12] Lucas, Robert E., Jr., 1988, "Money Demand in the United States: A Quantitative Review", *Carnegie-Rochester Conference Series on Public Policy,* Vol. 29, pp. 137-167

[13] Lucas, Robert E., Jr. and Stokey, Nancy L., 1983, "Optimal Fiscal and Monetary Policy in an Economy without Capital", *Journal of Monetary Economics,* Vol. 12, pp. 55-93

[14] Lucas, Robert E., Jr. and Stokey, Nancy L., 1987, "Money and Interest in a Cash-in-Advance Economy", *Econometrica,* Vol. 55, No. 9, pp. 491-513

[15] Marcet, Albert, 1991, "Solving Non-Linear Stochastic Models by Parameterizing Expectations: An Application to Asset Pricing with Production", Universitat Pompeu Fabra, Barcelona, Spain, mimeo

[16] McGrattan, Ellen R., 1998a, "Trends in Velocity and Policy Expectations, A Comment", *Carnegie-Rochester Conference Series on Public Policy,* Vol. 49, pp. 305-316

[17] McGrattan, Ellen R., 1998b, "A Defence of AK Growth Models", Federal Reserve Bank of Minneapolis, Quarterly Review, Fall, pp. 13-26

[18] Rodríguez-Mendizábal, Hugo, 1998, "The Variability of Money Velocity in a Generalized Cash-in-Advance Model", Working paper 320, Universitat Pompeu Fabra

[19] Svensson, Lars E. O., 1985, "Money and Asset Prices in a Cash-in-Advance Economy", *Journal of Political Economy,* Vol. 93, No. 5, pp. 919-944

Chapter 3

Money and Growth in a Cash-in-Advance Economy with Costly Credit

3.1 Introduction

This chapter studies how the specification of the payment that intermediaries charge for providing financial services affects the relationship between growth and the monetary system.

The question of money and growth can be addressed from two points of view, namely, to study the effect of money on growth, or the effect of growth on the monetary system. The first approach has been considered by many authors.[1] The second one has not been explored so extensively. One exception is the paper of Ireland (1994), in which both sides of the money-growth relationship are analyzed. He finds that the traditional effects of inflation on growth are small and the monetary system changes significantly due to the influence of growth.

Any mechanism that makes agents modify their holdings of real balances over time will lead to a change in the monetary system. For example, the increasing trend in income velocity is a consequence of the fact that in order to consume a constant proportion of output agents do not need to accumulate a constant proportion of output in the form of real balances. If agents are allowed to make a choice between 'cash' and

[1] Let us mention for example the theoretical contributions of Tobin (1965), Stockman (1981), Blackburn and Hung (1991), Jones and Manuelli (1995), Valentinyi (1995) and empirical ones by Roubini and Sala-i-Martin (1992), De Gregorio (1993), Gomme (1993).

'credit' goods, the evolution of the equilibrium cash-credit consumption ratio reflects the need or willingness of individuals to hold real balances. The cash-credit consumption ratio is a measure of how much of the total consumption is purchased using money. In traditional cash-credit models, e.g., Lucas (1984), Lucas and Stokey (1983, 1987), the cash-credit consumption ratio depends on the opportunity cost of holding money. The possibility of substitution between cash and credit consumptions gives rise to a decrease in real balances when the nominal interest rate increases. If some trends in velocity arise, they are due to trends in nominal interest rates.

Credit consumption in traditional cash-credit models can be viewed as a consumption with alternative means of payment with no cost of creation of money substitutes. However, creating money substitutes does not have to be necessarily free. For example, if a good is bought via a financial intermediary the buyer will have to bear a cost that the intermediary charges for providing financial services. Several papers formalize the idea that the monetary policy influences the decisions to devote resources to the creation of money substitutes, e.g., Gillman (1993), Aiyagari, Braun and Eckstein (1995). In these models the cash-credit consumption ratio changes due to changes in the nominal interest rates.

In Ireland (1994) money is driven out of the economy in the course of time due to the influence of growth. Resources to be paid to financial intermediaries for providing the alternative means of payment remain constant for all periods. Therefore, when the economy grows, it becomes relatively cheaper to consume via services of an intermediary, and individuals switch away from cash consumption towards 'costly' credit consumption. In the model of Ireland (1994) the cash-credit consumption ratio at each point of time depends on the nominal interest rates and on the levels of output achieved in the economy.

The goal of this chapter is to explore to what extension the payment to the intermediary affects the creation of money substitutes when dealing with the economy where the rate of growth is positive. We analyze the money-growth relationship, particularly the effects of growth on the monetary system. We show that the intermediation cost can alter the effects which growth has on the monetary system.

In our model money is introduced via a cash-in-advance constraint. Cash goods are paid using money whereas credit goods are paid via services of an intermediary. Intermediaries will be modeled as agents that provide financial services to the households charging some price. We will consider three different kinds of intermediation cost: proportional to the purchases made via an intermediary, non-proportionally related to the purchases made via an intermediary, and fixed. For the first case we obtain results that are analogous to the ones found in traditional cash-credit models, that is, when the intermediation cost increases proportionally to credit purchases, we do not observe any influence of growth on the monetary system. Under the non-proportional and fixed intermediation costs growth yields a transformation of the monetary system, what means that money is relatively driven out of the economy as the economy grows. The effect is stronger for the fixed intermediation cost.

In this context, even a model with AK technology might exhibit transition. We are interested in the effects of technology shocks and the government monetary policy both on the long run and on the short run equilibrium behavior. Changes in the monetary system are evaluated using as a measure the cash-credit consumption ratio, the income velocity of money and the volatility of real balances. When the monetary system changes by the influence of growth, the cash-credit consumption ratio decreases, income velocity of money increases and the volatility of real balances decreases over time. Higher nominal interest rates accelerate such an effect of economic growth.

The remainder of the chapter is organized as follows. The model and its main properties are described in section 3.2. In section 3.3 the equilibrium behavior of the economy is discussed and numerical simulations are performed for the cases of proportional, non-proportional and fixed intermediation costs. Final conclusions are summarized in section 3.4.

3.2 Model Description

3.2.1 The Household Problem

The economy consists of a large number of infinitely lived households. All households have identical preferences, production and trade opportunities.

Households inhabit the following environment: they face continuum of spatially separated markets, which are indexed by $i \in [0, 1]$. All households live in market 0, and the index i indicates the distance from home. In each market i a distinct perishable good is produced and sold in every period. Goods are thus indexed by i, which corresponds to the market of both production and trade. The economy has a representative household with preferences given by the utility function

$$E_t\left\{\sum_{j=t}^{\infty}\beta^{j-t}\int_0^1\frac{c_j(i)^{1-\theta}-1}{1-\theta}di\right\} \tag{3.1}$$

where $c_j(i)$ is defined as the consumption at period j of the good produced in market i and $\theta > 0$ is the inverse of the elasticity of intertemporal substitution.

The production and trade is like in Lucas and Stokey (1983). Each household is composed of a worker-shopper pair. At the beginning of every period agents learn the state of the economy. A representative worker decides to produce on any of the markets i via the net production function

$$y_t = A_t k_t \tag{3.2}$$

where A_t is a technology shock common in all markets. We assume that the logarithm of technology shocks follows an autoregressive process

$$\ln A_t = (1-\rho_A)\ln\bar{A} + \rho_A \ln A_{t-1} + \varepsilon_{A,t}$$

where ρ_A is the coefficient of correlation, $0 < \rho_A < 1$, $\bar{A}$ is the steady state level of the technology and the perturbations $\varepsilon_{A,t}$ are normally identically independently distributed, $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$.

Prior to any trading monetary holdings of agents are augmented by a lump sum transfer X_t from the government, which is endogenously determined in the system according to the current state of the economy and the nominal interest rate so that the money demand is totally satisfied. Government fixes the gross nominal interest rate R to be constant in all periods.[2] We will assume that $R > 1$.

Then the securities market opens. During the securities trading session households choose their currency holdings M_t. They also purchase (or issue) one-period nominally denominated pure discount bonds paying B_t units of money at period $t+1$ while they cost $\frac{B_t}{R}$ units of money at period t. Bonds are in zero net supply. Financial exchange is subject to the wealth constraint

$$\frac{M_t}{p_t} + \frac{B_t}{Rp_t} \leq w_t + \frac{X_t}{p_t}, \tag{3.3}$$

[2] As a policy instrument the government can target either the monetary aggregates or the interest rates. In this work the second approach will be used. This is because recently central banks have reoriented their operating procedures focusing on interest rates rather than regulating monetary aggregates. On the differences caused by targeting interest rates instead of monetary aggregates see for example Canzoneri and Dellas (1998), Collard, Dellas and Ertz (1998) or the chapter 4 of this dissertation.

where w_t is the real wealth at the beginning of period t. The real wealth at period $t = 1$ is given by the initial condition on nominal balances and bonds, and the initial price as

$$w_1 = \frac{M_0 + B_0}{p_1}.$$

The real wealth evolves according to

$$\begin{aligned} w_{t+1} &= \frac{p_t}{p_{t+1}} A_t k_t + \frac{M_t - p_t \int_0^1 [1 - \xi_t(i)]\, c_t(i) di}{p_{t+1}} + \\ & \frac{B_t - p_t \int_0^1 \xi_t(i) \left[c_t(i) + \gamma_t(i)\right] di}{p_{t+1}} - \frac{p_t}{p_{t+1}} k_{t+1}, \end{aligned} \tag{3.4}$$

where $\xi_t(i)$ is a parameter which takes values 0 or 1 according to if a good is purchased on market i with cash or issuing private securities, respectively. If a good is purchased on market i using private securities, agents employ intermediary's services and $\gamma_t(i)$ is the payment to the intermediary. The terms on the right hand side of (3.4) are: $p_t A_t k_t$ is the value of output carried from period t to $t + 1$, $M_t - p_t \int_0^1 [1 - \xi_t(i)]\, c_t(i)di$ is the excess currency that was accumulated during securities trading in t but not spent during shopping session, $B_t - p_t \int_0^1 \xi_t(i) \left[c_t(i) + \gamma_t(i)\right] di$ is the overall new debt issued in period t, where the beginning of period debt is augmented by the purchase of goods via services of an intermediary and $p_t k_{t+1}$ is the value of the capital to be saved and used for the $t + 1$ period production. After trading of the securities the production takes place and y_t is realized.

Finally, the goods market opens and consumption takes place. Worker stays at the market i during the whole period. Shopper visits various markets to acquire consumption goods carrying all the monetary balances of the household. At the end of the period both the shopper and the worker return to market zero.

Two ways of acquiring consumption goods are allowed: using money or issuing private securities. All goods purchased with government issued money will be referred to as cash goods. Goods purchased via services of an intermediary will be referred to as credit goods.

Nominal monetary balances M_t can be used to buy goods in some of the markets indexed by i. Cash purchases are subject to the liquidity constraint

$$\int_0^1 [1 - \xi_t(i)]\, c_t(i) di \leq \frac{M_t}{p_t}, \tag{3.5}$$

where $\xi_t(i) = \begin{cases} 0 \text{ if a good is purchased on market } i \text{ with cash,} \\ 1 \text{ if a good is purchased on market } i \text{ with privately issued securities.} \end{cases}$

As we have said, agents can issue private securities and pay for the consumption good through the intermediary. A financial intermediary provides financial services at a cost that is given for each market i and period t. Such a cost corresponds to the availability of intermediation services, checking the identity of buyers and their ability to pay. The seller (the worker) is willing to accept the private securities in exchange for the produced goods only because the intermediary guarantees that they will be paid for. Otherwise, the seller would only accept cash.[3] As the communication becomes more difficult when the shopper is far away from home (market zero), the payment to the intermediary increases with i. The real payment made to the intermediary is characterized by a function $\gamma_t(i)$ which is defined in the following way:

$$\gamma_t(i) = \gamma(i) \ \left[\xi_t(i) \ c_t(i)\right]^{\omega}, \text{ where } \omega \in [0,1]. \tag{3.6}$$

The time independent part of the payment, $\gamma(i)$, increases with the distance from home, and for the sake of tractability we assign it the following functional form:

$$\gamma(i) = \frac{i}{1-i}.$$

The time dependent part of the intermediation cost, $\left[\xi_t(i) \ c_t(i)\right]^{\omega}$, implies that for $\omega \in (0,1)$ it is proportionally more expensive to use the services of an intermediary for small shoppings, but it becomes relatively cheaper as the shoppings turn to be larger. When $\omega = 1$ the intermediation cost becomes proportional to purchases via services of an intermediary. For $\omega = 0$ the intermediation cost only depends on the distance.

The value of the cash consumption in the period t will be $p_t c_t(i)$ and the value of the credit consumption will be $p_t\left[c_t(i) + \gamma_t(i)\right]$. The budget constraint agents are facing can be written combining the wealth constraint (3.3) and the evolution of wealth of a household (3.4),

$$\frac{p_{t-1}}{p_t}\int_0^1 \left[c_{t-1}(i) + \xi_{t-1}(i)\gamma_{t-1}(i)\right] di + \frac{p_{t-1}}{p_t}k_t + \frac{M_t}{p_t} + \frac{B_t}{Rp_t} \ \leq \tag{3.7}$$

$$\frac{p_{t-1}}{p_t}A_{t-1}k_{t-1} + \frac{M_{t-1}}{p_t} + \frac{B_{t-1}}{p_t} + \frac{X_t}{p_t}.$$

[3]This is similar to the setup of Lucas (1984) where cash goods will be employed at the markets where the buyer is unknown to the seller, so the latter is unwilling to accept as payment the claims issued by the buyer. In the present model, however, the private securities will be accepted in all markets, but the payment to the intermediary may be so high that buying with cash might become preferred in some markets.

3.2.2 Equilibrium

A representative household chooses the stochastic sequences $\{c_t, k_{t+1}, M_t, B_t, \xi_t\}_{t=1}^{\infty}$ maximizing the expected discounted utility (3.1) subject to the budget constraint (3.7) and the cash-in-advance constraint (3.5).

Let λ_t and η_t be the non-negative Lagrange multipliers associated with the budget constrain (3.7) and the cash-in-advance constraints (3.5), respectively. The equations that characterize the equilibrium are the first order conditions on consumption, capital, nominal balances and nominal bonds, respectively,

$$
\begin{aligned}
c_t(i)^{-\theta} &= \beta E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right) + \eta_t\left[1 - \xi_t(i)\right], && (3.8)\\
\beta^2 E_t\left(\frac{\lambda_{t+2}p_{t+1}}{p_{t+2}}A_{t+1}\right) &= \beta E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right), && (3.9)\\
\eta_t &= \lambda_t - \beta E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right), && (3.10)\\
\frac{\lambda_t}{R} &= \beta E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right) && (3.11)
\end{aligned}
$$

and the following transversality conditions:

$$
\begin{aligned}
\lim_{j\to\infty} E_t\left(\beta^{t+j}\lambda_{t+j}k_{t+j+1}\right) &= 0, && (3.12)\\
\lim_{j\to\infty} E_t\left(\beta^{t+j}\lambda_{t+j}\frac{M_{t+j}}{p_{t+j}}\right) &= 0, && (3.13)\\
\lim_{j\to\infty} E_t\left(\beta^{t+j}\lambda_{t+j}\frac{B_{t+j}}{Rp_{t+j}}\right) &= 0. && (3.14)
\end{aligned}
$$

Definition: Given the set of initial conditions k_1, M_0, B_0, p_1 and the nominal interest rate R, the equilibrium consists of stochastic processes $\{c_t, k_{t+1}, M_t, B_t, \xi_t, X_t, p_t\}_{t=1}^{\infty}$ such that

(a) a representative household is maximizing the expected discounted utility (3.1) subject to the budget constraint (3.7) and the cash-in-advance constraint (3.5) choosing the stochastic sequences $\{c_t, k_{t+1}, M_t, B_t, \xi_t\}_{t=1}^{\infty}$,

(b) markets for goods, money and bonds clear in every period,

$$A_t k_t = \int_0^1 [c_t(i) + \xi_t(i)\gamma_t(i)]\, di + k_{t+1}, \tag{3.15}$$

$$M_t = M_{t-1} + X_t, \tag{3.16}$$

$$B_t = 0. \tag{3.17}$$

In what follows we describe how the decision on $\xi_t(i)$ is reached and what are its implications. Define

$$c_t(i) = \begin{cases} c_t^0(i) & \text{when } \xi_t(i) = 0, \\ c_t^1(i) & \text{when } \xi_t(i) = 1, \end{cases}$$

where $c_t^0(i)$ and $c_t^1(i)$ are the functions that characterize the cash and credit consumption per market i, respectively. Having defined the functions $c_t^0(i)$ and $c_t^1(i)$, using (3.10) and (3.11), we can rewrite the first order condition on consumption (3.8) as follows:

$$c_t^0(i)^{-\theta} = \lambda_t, \tag{3.18}$$

$$c_t^1(i)^{-\theta} = \frac{\lambda_t}{R}. \tag{3.19}$$

Consumers decide on the means of payment depending on the surplus they get. Such a surplus is

$$\frac{c_t^0(i)^{1-\theta} - 1}{1-\theta} - \lambda_t c_t^0(i) \quad \text{when buying with cash, and} \tag{3.20}$$

$$\frac{c_t^1(i)^{1-\theta} - 1}{1-\theta} - \frac{\lambda_t}{R}\left[c_t^1(i) + \gamma_t(i)\right] \quad \text{when buying via an intermediary.} \tag{3.21}$$

The value of $\xi_t(i)$ is given according to the surplus obtained on the corresponding market

$$\xi_t(i) = \begin{cases} 0 & \text{if} \quad \dfrac{c_t^0(i)^{1-\theta} - 1}{1-\theta} - \lambda_t c_t^0(i) \geq \dfrac{c_t^1(i)^{1-\theta} - 1}{1-\theta} - \dfrac{\lambda_t}{R}\left[c_t^1(i) + \gamma_t(i)\right] \\ 1 & \text{otherwise.} \end{cases}$$

When the two surpluses (3.20) and (3.21) equal, agents will be indifferent between using cash or privately issued securities. We can define a function

$$\chi(R, \lambda_t) : (1, \infty) \times (0, \infty) \to (0, 1)$$

that maps nominal interest rates and marginal utilities of wealth into the interval of markets, such that

$$\gamma_t\left(\chi(R,\lambda_t)\right) = \frac{R}{\lambda_t}\left[\frac{c^1(R,\lambda_t)^{1-\theta}-1}{1-\theta} - \frac{c^0(R,\lambda_t)^{1-\theta}-1}{1-\theta}\right] + Rc^0(R,\lambda_t) - c^1(R,\lambda_t), \tag{3.22}$$

where we use (3.18) and (3.19), that is, the fact that $c_t^0(i) = c^0(R,\lambda_t)$ and $c_t^1(i) = c^1(R,\lambda_t)$ are functions of the nominal interest rate R and the marginal utility of wealth λ_t. Since $R > 1$, there exists at each time t a cutoff index $\chi(R,\lambda_t) \in (0,1)$, such that in all markets with indexes $i < \chi(R,\lambda_t)$ consumers will use privately issued securities and in all markets with indexes $i \geq \chi(R,\lambda_t)$ consumers will use cash to acquire the consumption goods. Taking into account the expressions (3.18), (3.19), and (3.6), the equilibrium on goods market (3.15) can be rewritten as

$$A_t k_t = \int_{\chi(R,\lambda_t)}^{1} c^0(R,\lambda_t)di + \int_0^{\chi(R,\lambda_t)} c^1(R,\lambda_t)di + \int_0^{\chi(R,\lambda_t)} \frac{i}{1-i}c^1(R,\lambda_t)^{\omega}di + k_{t+1}. \tag{3.23}$$

The current period output is spent between cash consumption, credit consumption, payment to the intermediary and what is left is saved as the next period capital. The real monetary balances, which equal the amount of cash consumption purchased in all markets, are

$$m_t = \left[1-\chi(R,\lambda_t)\right] c^0(R,\lambda_t), \tag{3.24}$$

where

$$m_t = \frac{M_t}{p_t}. \tag{3.25}$$

The consumption via financial intermediaries, which equal the amount of credit consumption purchased in all markets, is

$$\varphi_t = \chi(R,\lambda_t)\ c^1(R,\lambda_t).$$

The payment to the intermediary is

$$\Gamma_t = \left\{-\chi(R,\lambda_t) - \ln\left[1-\chi(R,\lambda_t)\right]\right\}\ c^1(R,\lambda_t)^{\omega},$$

which represents the resources paid to the intermediary in all markets, where the term in the brackets is obtained solving the third integral in equation (3.23).

Let us now describe the equilibrium behavior of prices. From (3.11) it is clear that prices must be proportional to the marginal utility of wealth. We can conjecture a solution for prices in the form

$$p_t = \kappa_t \lambda_t, \tag{3.26}$$

where κ_t is a non-stochastic proportionality factor, which is just potentially time dependent. Plugging (3.26) into the first order condition on bonds (3.11), we get the evolution of the proportionality factor

$$\frac{\kappa_{t+1}}{\kappa_t} = \beta R. \tag{3.27}$$

The proportionality factor in the price function is therefore influenced by a government monetary policy, i.e., by the level of the nominal interest rate. Combining the equations (3.9) and (3.11) we can express the nominal interest rate to get the following Fisher relation between nominal and real interest rates, as in Sargent (1987):

$$\frac{1}{R} = \frac{1}{\rho_t} E_t \left(\frac{p_t}{p_{t+1}} \right) + \beta \mathrm{cov}_t \left(\frac{\lambda_{t+1}}{\lambda_t}, \frac{p_t}{p_{t+1}} \right), \tag{3.28}$$

where

$$\frac{1}{\rho_t} = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \tag{3.29}$$

is the real gross interest rate on risk free bonds. The nominal interest rate depends on the covariance between the growth rate of the marginal utility of wealth and the technology shocks. The covariance in (3.28) is negative, and the ratio of nominal and real interest rates exceeds the inverse of the expected rate of appreciation of money.

The monetary transfer from the government is determined endogenously combining expressions (3.16), (3.18), and (3.26), and it becomes a function of the marginal utility of wealth, nominal interest rate, proportional factor in prices and previous period nominal money holdings,

$$X_t = \kappa_t \left[1 - \chi(R, \lambda_t) \right] \lambda_t^{\frac{\theta - 1}{\theta}} - M_{t-1}.$$

3.3 Discussion of the Equilibrium Behavior and Numerical Simulations

Let us analyze the equilibrium behavior of the economy for the intermediation cost defined by the function (3.6),

$$\gamma_t(i) = \frac{i}{1-i}\, c^1(R, \lambda_t)^{\omega}. \tag{3.30}$$

Combining (3.30) and (3.22) we can write the cutoff index as

$$\chi(R, \lambda_t) = \frac{\left(\frac{\theta}{1-\theta}\right)\left(R^{\frac{1-\theta}{\theta}} - 1\right)R}{\lambda_t^{\frac{1-\omega}{\theta}} R^{\frac{\omega}{\theta}} + \left(\frac{\theta}{1-\theta}\right)\left(R^{\frac{1-\theta}{\theta}} - 1\right)R}. \tag{3.31}$$

Note that the cutoff index depends on the marginal utility of wealth and the nominal interest rate. We can see from (3.18)-(3.19) that consumption and marginal utility of wealth are inversely related. When the consumption grows to infinity, the marginal utility of wealth λ_t decreases to zero. The proportion of markets in which agents employ services of an intermediary is given by the cutoff index.

We will now state the expressions for variables that give us some information about the effect of growth on the monetary system: real money demand, cash-credit consumption ratio and income velocity of money. Later on we will analyze in more detail the behavior of these variables for particular intermediation costs, i.e., for different values of the parameter ω. The real money demand (cash consumption) is given by the equation (3.24) and can be rewritten plugging in the expressions for the cutoff index (3.31) and for the cash consumption per market (3.18) as follows:

$$m_t = \frac{\left(\frac{R}{\lambda_t}\right)^{\frac{\omega}{\theta}}}{\lambda_t^{\frac{1-\omega}{\theta}} R^{\frac{\omega}{\theta}} + \left(\frac{\theta}{1-\theta}\right)\left(R^{\frac{1-\theta}{\theta}} - 1\right)R}. \tag{3.32}$$

The cash-credit consumption ratio can be obtained combining (3.19), (3.31) and (3.32),

$$\frac{m_t}{\varphi_t} = \frac{\lambda_t^{\frac{1-\omega}{\theta}}}{\left(\frac{\theta}{1-\theta}\right)\left(R^{\frac{1-\theta}{\theta}} - 1\right)R^{\frac{1+\theta-\omega}{\theta}}}. \tag{3.33}$$

When the cash-credit consumption ratio decreases, money is substituted by services of an intermediary.

Proposition 1 *The cash-credit consumption ratio diminishes with the increasing nominal interest rate, i.e., the derivative* $\frac{\partial}{\partial R}(m_t/\varphi_t)$ *is negative.*

Proof. See Appendix 1. ■

The income velocity of money is given as the ratio of output (3.2) to real monetary balances (3.32)

$$v_t = A_t k_t \, \lambda_t^{\frac{1}{\theta}} + \left(\frac{\theta}{1-\theta}\right)\left(R^{\frac{1-\theta}{\theta}} - 1\right) R^{\frac{\theta-\omega}{\theta}} A_t k_t \, \lambda_t^{\frac{\omega}{\theta}}. \tag{3.34}$$

Proposition 2 *Income velocity of money increases with increasing nominal interest rate, i.e., the derivative* $\frac{\partial \, v_t}{\partial R}$ *is positive.*

Proof. Using the same argument as the one in the proof of Proposition 1. ■

There are some results that can be developed analytically, but they do not highlight all properties of the model. Therefore, we simulate the economy applying the method of parameterized expectations developed in Den Haan and Marcet (1990). An application of the method in a monetary model can be found for example in Den Haan (1994). In Appendix 2 we describe in more detail the system of equilibrium equations and the particular steps which must be performed to reach the solution in our model.

In order to analyze the fluctuations of some endogenous variables we define the volatility of a variable h_t, σ_h, as a standard error of the residuals of the regression

$$\ln h_t = \zeta_h + \rho_h \ln h_{t-1} + \varepsilon_{h,t}.$$

The values of the parameters are specified as following: the discounting factor $\beta = 0.99$, the correlation coefficient $\rho_A = 0.95$, the standard error of technology shocks $\sigma_A = 0.007$, like in Cooley and Prescott (1995), and $\bar{A} = 1.036$. The steady state value of the technology is set in order to reach an average growth rate around 2-4% per period. That implies that one period in the model corresponds to one year.

3.3.1 Proportional Intermediation Cost

In this section we will consider the case in which the intermediation cost varies proportionally to the purchases realized via services of an intermediary, that is, we set $\omega = 1$ in equation (3.30). When agents use the services of an intermediary they have to pay an extra charge which represents a given percentage of their shoppings.

We see that the cutoff index (3.31) is independent of the technology shock A_t. Therefore, the portion of markets in which services of an intermediary or cash are employed varies only due to changes in the monetary policy. As the economy grows, the real money demand (3.32) tends to infinity and the ratio between cash and credit consumptions (3.33) will be dependent on the monetary policy only.

Proposition 3 *Cash consumption, credit consumption and the payment to the intermediary all grow at the rate* $(\lambda_t/\lambda_{t+1})^{\frac{1}{\theta}}$.

Proof. See Appendix 1. ■

Taking into account the goods market equilibrium (3.23), capital will also grow at the same rate. To find this growth rate we can calculate the policy function for capital. It can be obtained by solving the following dynamic programming problem

$$v(k_t, A_t) = \max_{k_{t+1}} \left\{ \frac{(A_t k_t - k_{t+1})^{1-\theta} - 1}{1-\theta} + \beta E\left[v(k_{t+1}, A_{t+1})\right] \right\}. \tag{3.35}$$

The policy function for the logarithmic utility function, $\theta = 1$, can be found analytically and it is

$$k_{t+1} = \beta A_t k_t. \tag{3.36}$$

The corresponding evolution of the marginal utility of wealth is then

$$\lambda_{t+1} = \frac{1}{\beta A_{t+1}} \lambda_t.$$

When $\theta \neq 1$, the solution must be found numerically.

The income velocity of money (3.34) in general varies with both technology shocks and with the nominal interest rate. Nevertheless, for the logarithmic utility function the relationship between the marginal utility of wealth and technology shocks is

$$\lambda_t = \frac{1}{(1-\beta)A_t k_t} \tag{3.37}$$

and the velocity in (3.34) only responds to changes in the nominal interest rate.

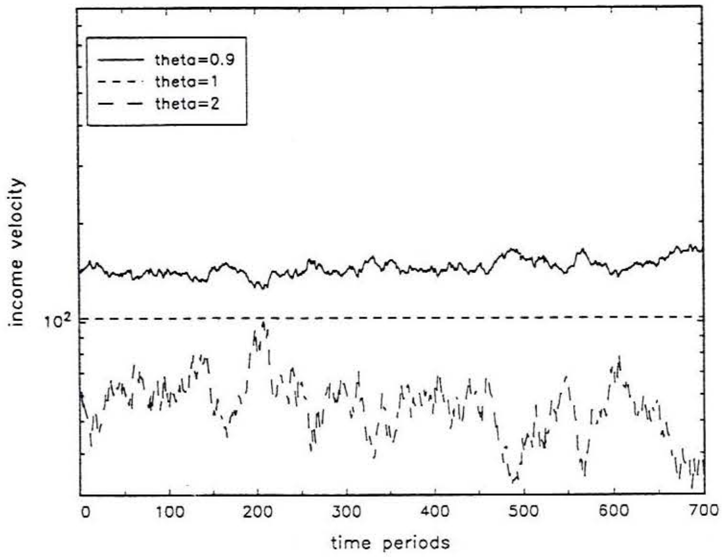

Figure 3.1: Income velocity of money for the proportional intermediation cost, different values of the elasticity of intertemporal substitution, and $R = 1.03$.

In Figure 3.1 we also compare the behavior of the income velocity of money for $\theta = 0.9$, 1 and 2 for $R = 1.03$. The volatility of the income velocity of money is affected by the elasticity of intertemporal substitution of agents. The standard error σ_v represents the volatility of the income velocity. We get the following values for such a standard error: $\sigma_v \mid_{\theta=0.9}= 0.0156$ and $\sigma_v \mid_{\theta=2}= 0.0654$.

For $\theta = 1$, the real money demand fluctuates as output, capital and credit consumption. The volatility of real balances is $\sigma_m \mid_{\theta=1}= 0.024$.

Summing up, under the proportional intermediation cost the payment to the intermediary remains constant in relative terms, i.e., when the economy grows, agents consumption grows, but they pay a given percentage of their credit purchases to the intermediary. The fraction of markets where services of an intermediary are employed remains constant, since the cutoff index only depends on the nominal interest rate R which is set constant. When the nominal interest rate increases, the fraction of markets in which it is cheaper to buy via intermediaries increases.

3.3.2 Non-Proportional Intermediation Cost

Let us analyze the behavior of the economy for the intermediation cost defined by the function (3.30) for $\omega \in (0,1)$. As the consumption grows, the marginal utility of wealth λ_t decreases to zero. Therefore, in the limit as time goes to infinity the cutoff index $\chi(R, \lambda_t)$, given in (3.31), approaches unity. This means that the proportion of markets in which agents employ services of an intermediary increases in the course of time. The real money demand (3.32) increases to infinity and the cash-credit consumption ratio (3.33) decreases to zero as the economy grows. This implies that the long run growth rate of credit consumption is higher than the growth rate of cash consumption. The switch from cash towards credit consumption is stimulated both by growth (the marginal utility of wealth decreases as the economy grows) and by higher nominal interest rate (the cash-credit consumption ratio decreases as R increases).

Proposition 4 *Cash consumption and the payment to the intermediary in the long run grow at the rate* $(\lambda_t/\lambda_{t+1})^{\frac{\omega}{\theta}}$. *Credit consumption in the long run grows at the rate* $(\lambda_t/\lambda_{t+1})^{\frac{1}{\theta}}$.

Proof. See Appendix 1. ■

As we cannot say much more about the equilibrium behavior of the economy using only the analytical approach, we simulate the economy, setting $\omega = 0.5$.

First we present the behavior of capital and cash and credit consumptions. Monetary policy is set to $R = 1.03$ for all time periods and the utility function is taken to be logarithmic, i.e. $\theta = 1$. In Figure 3.2 we can observe the switch towards credit consumption in the course of time, $\chi(R, \lambda_t) \to 1$ as the economy grows. We can also see that the growth rate of cash consumption is in fact lower than the growth rate of credit consumption, both in the long and short run. In this case the volatility of real balances σ_m (cash consumption) in the long run is lower than the one of capital σ_k, $\sigma_m \mid_{\theta=1} = 0.013$ and $\sigma_k \mid_{\theta=1} = 0.024$.

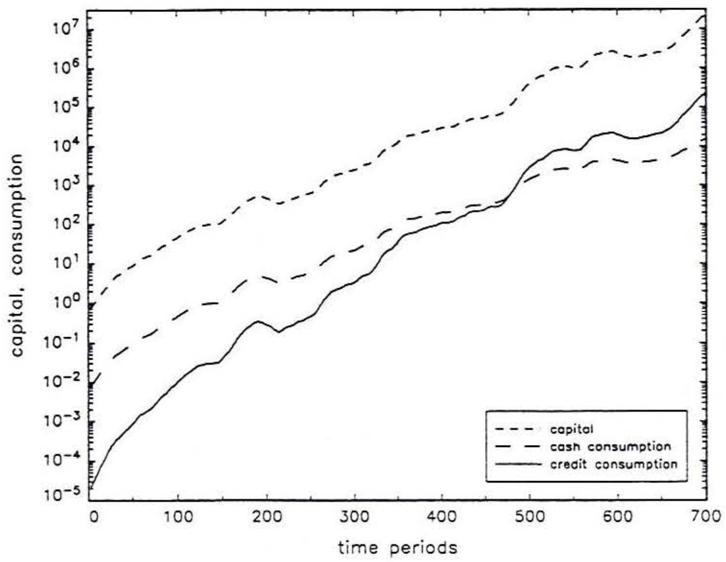

Figure 3.2: Equilibrium behavior of capital, and cash and credit consumptions for the non-proportional intermediation cost for $\omega = 0.5$, for the monetary policy $R = 1.03$ and the logarithmic utility function, $\theta = 1$.

Let us see now the effect of different monetary policies on the equilibrium behavior. We will compare the results for environments with different nominal interest rates, $R = 1.03$, 1.13 and 1.23. Figure 3.3 shows the real money demand for the corresponding monetary policies. When the nominal interest rate increases, money becomes less attractive to hold and the levels of the real money demanded decrease. However, in the long run the growth rates of the real monetary balances converge to the same value for different nominal interest rates. The behavior of credit consumption under different monetary policies is plotted in Figure 3.4. When higher interest rates are employed, credit consumption increases as the fraction of markets in which services of intermediaries are employed increases. In the long run the effect of monetary policy on credit consumption vanishes.

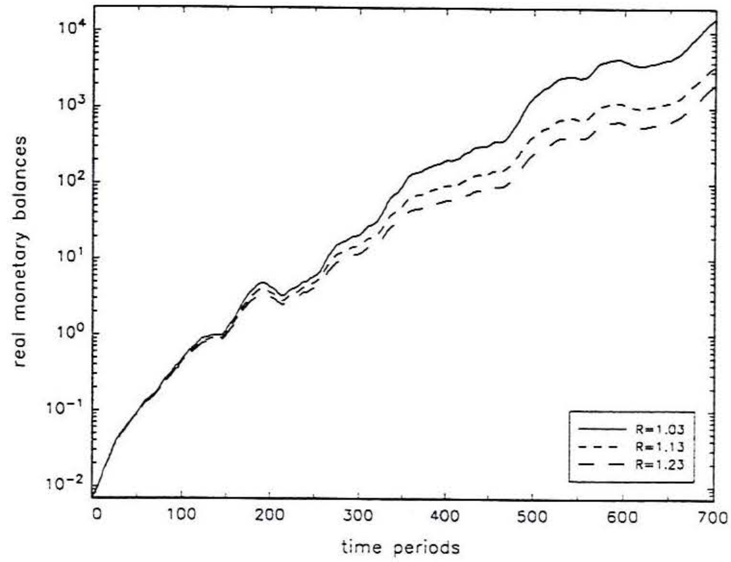

Figure 3.3: Equilibrium behavior of the real monetary balances for the non-proportional intermediation cost for $\omega = 0.5$, under three different monetary policies, $R = 1.03$, 1.13 and 1.23, and $\theta = 1$.

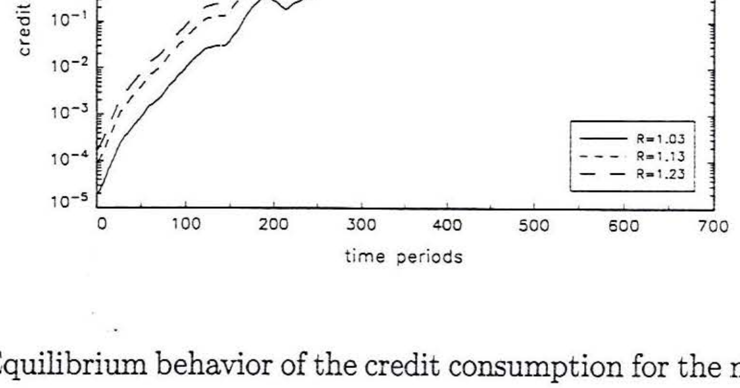

Figure 3.4: Equilibrium behavior of the credit consumption for the non-proportional intermediation cost for $\omega = 0.5$, under three different monetary policies, $R = 1.03, 1.13$ and 1.23, and $\theta = 1$.

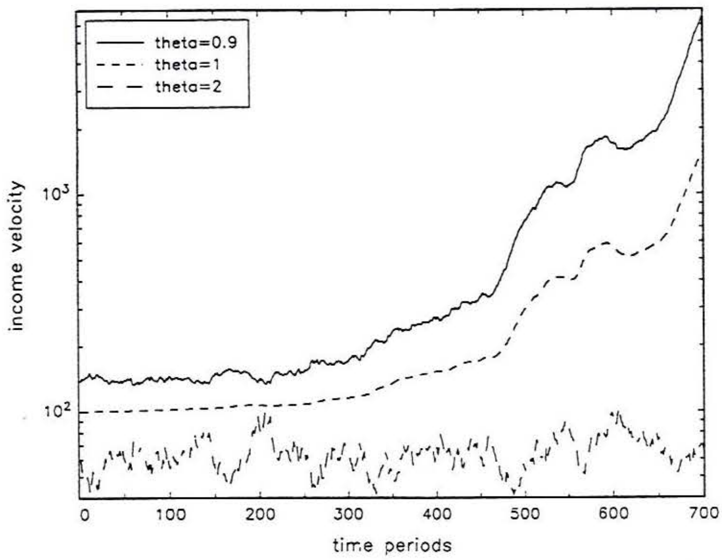

Figure 3.5: Income velocity of money for the non-proportional intermediation cost for $\omega = 0.5$, different values of the elasticity of intertemporal substitution, and $R = 1.03$.

Income velocity of money (3.34) will have an increasing trend in the long run if capital increases more rapidly than the real monetary balances. As can be seen in Figure 3.2, this holds for our case. We plot the behavior of the income velocity of money for $\theta = 0.9$, 1 and 2 and for $R = 1.03$ in Figure 3.5. The income velocity of money increases in the long run, as money is used for a lower fraction of consumption purchases. It is higher for higher nominal interest rates (see Proposition 2) what is a consequence of the lower money demand under higher nominal interest rates. The volatility of the income velocity changes with the elasticity of intertemporal substitution, similarly as in the case of the proportional intermediation cost.

From the nature of the intermediation cost we know that for higher purchases it becomes cheaper to consume via services of an intermediary. At early times, cash consumption is employed in a higher fraction of the markets because it is expensive to consume via intermediaries. Later on, as the consumption grows, it is cheaper to buy via services of an intermediary in more markets. The fraction of markets where cash is employed decreases. Both consumptions grow at positive rates, however the growth rate of credit consumption is higher. We thus see that for the non-proportional intermediation cost growth has an effect on the monetary system. Money is relatively less used as the economy grows.

3.3.3 Fixed Intermediation Cost

We now proceed with the analysis of the case in which the cost of providing financial services does not depend on the level of credit purchases. The payment for using financial services only depends on the distance from home. This means that we set $\omega = 0$ in (3.30).

The cutoff index $\chi(R, \lambda_t)$ approaches 1 as time goes to infinity. Similarly as in the previously analyzed case, agents employ services of an intermediary in a higher proportion of markets as the economy grows. Technology shocks only affect the real money demand through the marginal utility of wealth. As λ_t decreases to zero in the long run, the effect of technology shocks is negligible and the real money demand (3.32) varies only with the nominal interest rate. The ratio between cash and credit consumptions decreases to zero in the course of time. As before, the switch from cash to credit consumption is stimulated by growth and higher opportunity cost of holding money.

Proposition 5 *Cash consumption and the payment to the intermediary in the long run grow at the rate one, i.e., they become stationary. Credit consumption in the long run grow at the rate* $(\lambda_t/\lambda_{t+1})^{\frac{1}{\theta}}$.

Proof. See the proof of the Proposition 4, in Appendix 1 and set $\omega = 0$. ■

The long run growth rate of credit consumption will approach the growth rate of capital. This can be seen from the goods market equilibrium condition (3.23), where the only terms growing as $t \to \infty$ are the credit consumption and capital. The cash consumption and the payment to the intermediary do not grow in the long run, as stated in the Proposition 5. The long run policy function of capital can be calculated solving the problem (3.35) and finding the policy function of a non-monetary economy.

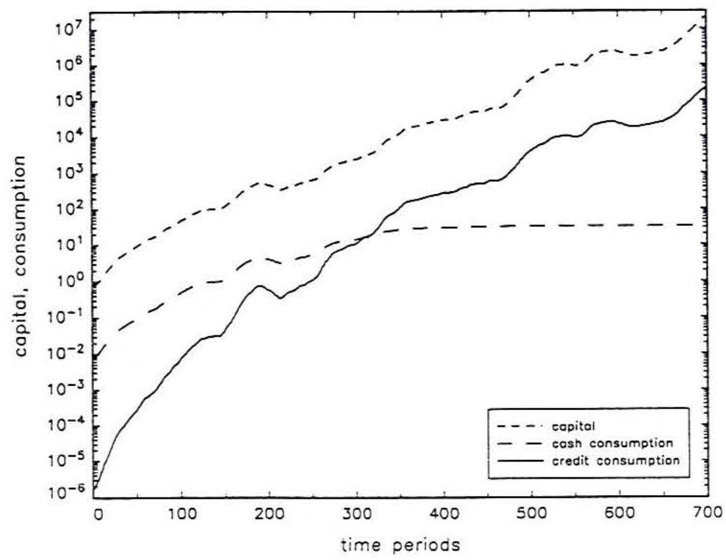

Figure 3.6: Equilibrium behavior of capital, and cash and credit consumptions for the fixed intermediation cost, for the monetary policy $R = 1.03$ and the logarithmic utility function, $\theta = 1$.

We present the behavior of capital and cash and credit consumptions. Monetary policy is specified as $R = 1.03$ for all time periods. The results reported in Figure 3.6 are calculated for the logarithmic utility function, for $\theta = 1$. We can see the behavior of the economy which we have already described intuitively. Households shift towards the purchases via services of an intermediary. When the levels of output are high enough, credit consumption grows like capital and the demand for real monetary balances saturates and does not fluctuate any more. The long run property of real money demand thus emerges: it becomes only interest elastic. Once the nominal interest rate is set the long run real money demand becomes perfectly predictable. As a consequence of economic growth, agents switch from cash to credit consumption for any level of the nominal interest set by the government, and the behavior of the economy approaches the one of a non-monetary economy. The monetary policy just affects the speed of transformation, as $\frac{\partial}{\partial R}(m_t/\varphi_t) < 0$ (see Proposition 1).

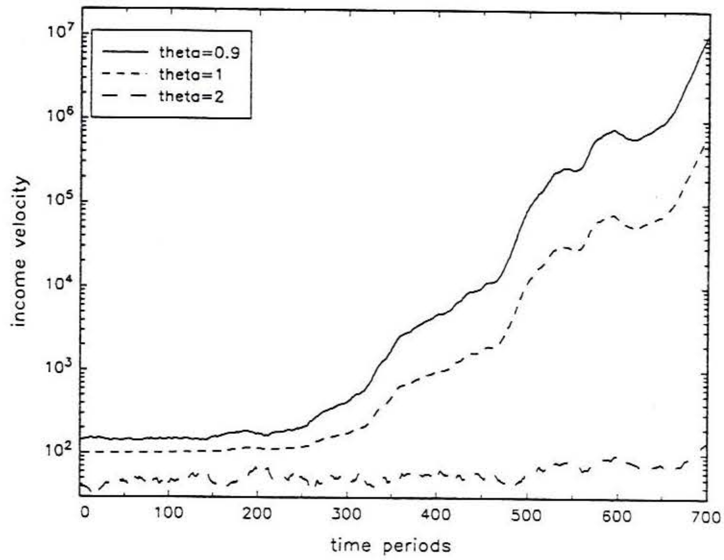

Figure 3.7: Income velocity of money for the fixed intermediation cost, different values of the elasticity of intertemporal substitution, and $R = 1.03$.

We must note that when the economy is richer at the beginning, i.e., the level of initial capital is higher, the switch towards consuming via services of an intermediary is observed sooner. Thus an economy with sufficiently high initial capital could behave like a non-monetary one from the very beginning.

In Figure 3.7 we plot the behavior of the income velocity for $\theta = 0.9$, 1 and 2 for $R = 1.03$. We observe an analogous behavior as the one for the proportional intermediation cost for early periods. However, in later periods, as the money is relatively driven out from the economy, the income velocity of money increases.

Summing up, when the intermediation cost is fixed, it becomes relatively cheaper to use the services of an intermediary as the economy becomes richer. In the long run credit consumption is employed almost in all markets. Even if money does not disappear from the economy, only a negligible fraction of total consumption is acquired in exchange for money and we end up dealing with a non-monetary economy in relative terms.

3.4 Conclusion

It has been shown that the behavior of financial intermediaries may influence the relationship that arises between money and growth. Different specifications of the intermediation cost affect the performance of the economy. We have analyzed the influence of technology shocks and nominal interest rates on the equilibrium behavior. A particular attention has been paid to the behavior of real balances, cash-credit consumption ratio and income velocity of money.

When the proportional intermediation cost is considered, the monetary system of the economy does not experience any changes in the course of time. Higher nominal interest rate just increases the number of markets where agents purchase via services of an intermediary.

When the non-proportional intermediation cost is introduced, we observe an influence of growth on the monetary system. The properties of the model imply that agents consume using money always when the services of an intermediary are relatively expensive. When the economy grows, services of an intermediary become relatively cheaper and agents switch from cash towards credit consumption. That causes that money is relatively less used as the economy grows.

The fixed intermediation cost in an extreme case of the non-proportional intermediation cost. Due to the influence of growth, in the long run money is used only for a negligible fraction of consumption purchases.

Comparing all cases analyzed in this chapter we see that the intermediation cost may influence the magnitude of the effect that growth has on the monetary system. As the parameter in the intermediation cost ω decreases towards zero, the effect of growth on the monetary system becomes stronger and money is driven out of the economy more rapidly. Concerning the fluctuations of endogenous variables we observe that in the long run, output, capital and credit consumption have all the same volatility and it is independent of the specification of the intermediation cost. The volatility of real balances in the long run in general differs from the ones of output, capital and credit consumption. Fluctuations of real monetary balances in the long run are dependent on the parameter ω. Increasing ω makes the volatility of real balances in the long run decrease. This effect is the strongest in the case of the fixed intermediation cost, when the government can control the equilibrium long run real money demand just by specifying the nominal interest rate. Higher nominal interest rates accelerate the transformation of the economy towards a non-monetary one.

3.5 Appendix

Appendix 1

Proof of the Proposition 1:

The derivative of m_t/φ_t with respect to the nominal interest rate is

$$\frac{\partial}{\partial R}\left(\frac{m_t}{\varphi_t}\right)=\begin{cases}\dfrac{\lambda_t^{1-\omega}\left[(\omega-2)\ln R-1\right]}{R^{3-\omega}(\ln R)^2} & \text{for } \theta=1, \text{ and}\\[2ex] \left(\dfrac{1-\theta}{\theta}\right)\lambda_t^{\frac{1-\omega}{\theta}}\dfrac{R^{\frac{\omega-1-2\theta}{\theta}}\left[R^{\frac{1-\theta}{\theta}}\left(\dfrac{\omega-2}{\theta}\right)-\dfrac{\omega-1-\theta}{\theta}\right]}{\left(R^{\frac{1-\theta}{\theta}}-1\right)^2} & \text{for } \theta\neq 1.\end{cases} \tag{3.38}$$

To analyze the sign of this expression we have to consider three cases:

a) for $\theta = 1$, the expression (3.38) is negative, because $\omega \leq 1$ and the term $[(\omega-2)\ln R - 1]$ is negative;

b) for $\theta < 1$, the expression (3.38) is negative when $R^{\frac{1-\theta}{\theta}} > \frac{\omega-1-\theta}{\omega-2}$. Such an inequality holds, since $R > 1$ implies that $R^{\frac{1-\theta}{\theta}} > 1$, and $\omega - 1 - \theta < \omega - 2$;

c) for $\theta > 1$, the expression (3.38) is negative when $R^{\frac{1-\theta}{\theta}} < \frac{\omega-1-\theta}{\omega-2}$. Such an inequality holds, since $R^{\frac{1-\theta}{\theta}} < 1$, and thus $\frac{\omega-1-\theta}{\omega-2} > 1$.

Proof of the Proposition 3:

The growth rate of cash consumption $g_t^m\mid_{\omega=1}$ in a model with proportional intermediation cost is

$$g_t^m\mid_{\omega=1}=\frac{\left[1-\chi(R,\lambda_{t+1})\right]c^0(R,\lambda_{t+1})}{\left[1-\chi(R,\lambda_t)\right]c^0(R,\lambda_t)},$$

the growth rate of credit consumption $g_t^\varphi\mid_{\omega=1}$ is

$$g_t^\varphi\mid_{\omega=1}=\frac{\chi(R,\lambda_{t+1})\ c^1(R,\lambda_{t+1})}{\chi(R,\lambda_t)\ c^1(R,\lambda_t)}$$

and the growth rate of the payment to the intermediary $g_t^\Gamma\mid_{\omega=1}$ is

$$g_t^\Gamma\mid_{\omega=1}=\frac{\left\{-\chi(R,\lambda_{t+1})-\ln\left[1-\chi(R,\lambda_{t+1})\right]\right\}c^1(R,\lambda_{t+1})}{\left\{-\chi(R,\lambda_t)-\ln\left[1-\chi(R,\lambda_t)\right]\right\}c^1(R,\lambda_t)}.$$

Plugging the expressions for consumptions and the cutoff index, (3.18), (3.19) and (3.31), into these growth rates we get that all three variables grow at the common rate

$$g_t^m \mid_{\omega=1} = g_t^\varphi \mid_{\omega=1} = g_t^\Gamma \mid_{\omega=1} = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{1}{\theta}}.$$

Proof of the Proposition 4:

The growth rates of cash consumption g_t^m and credit consumption g_t^φ in a model with the non-proportional intermediation cost can be expressed analogously to the case of the proportional intermediation cost specification,

$$g_t^m = \frac{[1-\chi(R,\lambda_{t+1})]\, c^0(R,\lambda_{t+1})}{[1-\chi(R,\lambda_t)]\, c^0(R,\lambda_t)} \text{ and } g_t^\varphi = \frac{\chi(R,\lambda_{t+1})c^1(R,\lambda_{t+1})}{\chi(R,\lambda_t)c^1(R,\lambda_t)}.$$

The analytical expressions for consumption growth rates are complicated to treat when looking at short run behavior. We can, however, analyze the long run behavior, i.e. when $\lambda_t \to 0$. The long run growth rate of cash consumption is

$$\lim_{t\to\infty} g_t^m = \lim_{t\to\infty}\left(\frac{\lambda_t^{\frac{\omega}{\theta}}\left[\lambda_t^{\frac{1-\omega}{\theta}} R^{\frac{\omega}{\theta}} + \left(\frac{\theta}{1-\theta}\right) R\left(R^{\frac{1-\theta}{\theta}}-1\right)\right]}{\lambda_{t+1}^{\frac{\omega}{\theta}}\left[\lambda_{t+1}^{\frac{1-\omega}{\theta}} R^{\frac{\omega}{\theta}} + \left(\frac{\theta}{1-\theta}\right) R\left(R^{\frac{1-\theta}{\theta}}-1\right)\right]}\right) = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{\omega}{\theta}}.$$

The long run growth rate of credit consumption is

$$\lim_{t\to\infty} g_t^\varphi = \lim_{t\to\infty}\left(\frac{\lambda_t^{\frac{1}{\theta}}\left[\lambda_t^{\frac{1-\omega}{\theta}} R^{\frac{\omega}{\theta}} + \left(\frac{\theta}{1-\theta}\right) R\left(R^{\frac{1-\theta}{\theta}}-1\right)\right]}{\lambda_{t+1}^{\frac{1}{\theta}}\left[\lambda_{t+1}^{\frac{1-\omega}{\theta}} R^{\frac{\omega}{\theta}} + \left(\frac{\theta}{1-\theta}\right) R\left(R^{\frac{1-\theta}{\theta}}-1\right)\right]}\right) = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{1}{\theta}}.$$

The long run growth rate of the payment to the intermediary is

$$\lim_{t\to\infty} g_t^\Gamma = \lim_{t\to\infty}\left(\frac{\{-\chi(R,\lambda_{t+1}) - \ln[1-\chi(R,\lambda_{t+1})]\}}{\{-\chi(R,\lambda_t) - \ln[1-\chi(R,\lambda_t)]\}}\frac{\lambda_t^{\frac{1}{\theta}}}{\lambda_{t+1}^{\frac{1}{\theta}}}\right) = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{\omega}{\theta}}.$$

Appendix 2

Solution Method

In order to apply the technique of parameterized expectations we transform the variables as follows:

$$\hat{c}_t^0 = \frac{c^0(R,\lambda_t)}{k_t},\ \hat{c}_t^1 = \frac{c^1(R,\lambda_t)}{k_t},\ \hat{k}_{t+1} = \frac{k_{t+1}}{k_t},\ \hat{m}_t = \frac{m_t}{k_t},\ \hat{\lambda}_t = \lambda_t k_t^\theta,\ \hat{p}_t = p_t k_t.$$

Then the equilibrium equations (3.18)-(3.19), (3.9), (3.22), (3.31)-(3.32), (3.23), (3.25)-(3.26) and (3.16)-(3.17) can be written as follows:

$$\hat{c}_t^0 = \left(\frac{1}{\hat{\lambda}_t}\right)^{\frac{1}{\theta}}, \tag{3.39}$$

$$\hat{c}_t^1 = \left(\frac{R}{\hat{\lambda}_t}\right)^{\frac{1}{\theta}}, \tag{3.40}$$

$$\hat{\lambda}_t = \beta E_t\left(\hat{\lambda}_{t+1}A_{t+1}\hat{k}_{t+1}^{-\theta}\right), \tag{3.41}$$

$$\chi(R,\lambda_t) = \begin{cases} \dfrac{R\ \ln R}{\lambda_t^{1-\omega}R^{\omega} + R\ \ln R} \text{ for } \theta = 1, \text{ and} \\[2em] \dfrac{\left(\dfrac{\theta}{1-\theta}\right)R\left(R^{\frac{1-\theta}{\theta}}-1\right)}{\lambda_t^{\frac{1-\omega}{\theta}}R^{\frac{\omega}{\theta}} + \left(\dfrac{\theta}{1-\theta}\right)R\left(R^{\frac{1-\theta}{\theta}}-1\right)} \text{ for } \theta \neq 1, \end{cases} \tag{3.42}$$

$$\hat{m}_t = [1-\chi(R,\lambda_t)]\,\hat{c}_t^0, \tag{3.43}$$

$$A_t = \hat{m}_t + \chi(R,\lambda_t)\hat{c}_t^1 + \hat{k}_{t+1} + \frac{\Gamma_t}{k_t}, \tag{3.44}$$

$$\text{where } \Gamma_t = \{-\chi(R,\lambda_t) - \ln[1-\chi(R,\lambda_t)]\}\ c^1(R,\lambda_t)^{\omega}, \tag{3.45}$$

$$\pi_1 = \frac{p_1}{\lambda_1},\ \pi_{t+1} = \beta R\pi_t, \tag{3.46}$$

$$\hat{p}_t = \pi_t\lambda_t k_t, \tag{3.47}$$

$$M_t = \hat{m}_t\hat{p}_t, \tag{3.48}$$

$$X_t = M_t - M_{t-1}, \tag{3.49}$$

$$B_t = 0 \tag{3.50}$$

where k_1, M_0, p_1 and R are given. The expression (3.41) can be obtained simplifying (3.9) by plugging in (3.26)-(3.27).

We will parameterize the expectation in (3.41) by a first degree polynomial

$$E_t\left(\hat{\lambda}_{t+1}A_{t+1}\hat{k}_{t+1}^{-\theta}\right) = \psi(A_t;b) = b_1 A_t^{b_2} \tag{3.51}$$

where $b = (b_1, b_2)'$ is the vector of parameters. We use the long run analytical solution for the fixed intermediation cost and $\theta = 1$ to assign the initial values to the vector of parameters, $b^0 = (b_1^0, b_2^0)'$. We know that the following holds for the growth rate of capital and the transformed marginal utility of wealth:

$$\begin{aligned} \hat{k}_{t+1} &= \beta A_t, \\ \hat{\lambda}_t &= \frac{1}{(1-\beta) A_t}. \end{aligned}$$

This implies that, in order to parameterize the expectation (3.51),

$$\hat{\lambda}_t = \beta b_1 A_t^{b_2} = \frac{1}{(1-\beta) A_t}$$

and we have to set $b^0 = (\frac{1}{\beta(1-\beta)}, -1)$.

To apply the algorithm we first have to simulate time series for the exogenous process $\{A_t\}$. The model can be solved for an initial value of the parameter vector b^0, and the time series for the endogenous process $\{\hat{\lambda}_t, \hat{c}_t^0, \hat{c}_t^1, \hat{k}_{t+1}, \chi_t, \hat{m}_t, \hat{p}_t, M_t, X_t\}_{t=1}^T$ can be obtained from equations (3.39)-(3.49). For any given value of b^0 a new value b^1 can be found by running a non-linear least squares regression on

$$\hat{\lambda}_{t+1} A_{t+1} \hat{k}_{t+1}^{-\theta}.$$

The standard implementation of the parameterized expectations algorithm finds the best approximation by iterating on this process. We iterate for 3000 observations until the precision of 3 digits is reached between two consecutive solutions for the parameter b.

Bibliography

[1] Aiyagari, S. Rao, Braun, Toni and Eckstein, Zvi, 1995, "Transaction Services, Inflation, and Welfare", Federal Reserve Bank of Minneapolis, Research Department, Working Paper 551

[2] Blackburn, Keith and Hung, Victor T. Y., 1991, "Money and Growth", Discussion paper in Economics and Econometrics No. 9110, University of Southampton

[3] Canzoneri, Matthew B. and Dellas, Haris, 1998, "Real Interest Rates and Central Bank Operating Procedures", *Journal of Monetary Economics,* Vol. 42, pp. 471-494

[4] Collard, Fabrice, Dellas, Harris and Ertz, Guy, 1998, "Poole Revisited", IRES, Université Catolique de Louvain, Discussion Paper 9823

[5] Cooley, Thomas F. and Prescott, Edward C., 1995, "Economic Growth and Business Cycles", in Frontier of Business Cycle Research, Ed. Thomas F. Cooley, Princeton University Press

[6] De Gregorio, José, 1993, "Inflation, Taxation, and Long-Run Growth", *Journal of Monetary Economics,* Vol. 31, pp. 271-298

[7] Den Haan, Wouter J., 1990, "The Optimal Inflation Path in a Sidrauski-Type Model with Uncertainty", *Journal of Monetary Economics,* Vol. 25, pp. 389-409

[8] Den Haan, Wouter J. and Marcet, Albert, 1990, "Solving the Stochastic Growth Model by Parameterizing Expectations", *Journal of Business and Economic Statistics,* Vol. 8, No. 1, pp. 31-34

[9] Gillman, Max, 1993, "The Welfare Cost of Inflation in a Cash-in-Advance Economy with Costly Credit", *Journal of Monetary Economics,* Vol. 32, pp. 51-77

[10] Gomme, Paul, 1993, "Money and Growth Revisited: Measuring the Costs of Inflation in and Endogenous Growth Model", *Journal of Monetary Economics,* Vol. 31, pp. 97-115

[11] Ireland, Peter N., 1994, "Money and Growth: An Alternative Approach", *American Economic Review,* Vol. 84, No. 1, pp. 47-65

[12] Jones, Larry E. and Manuelli, Rodolfo E., 1995, "Growth and the Effects of Inflation", *Journal of Economic Dynamics and Control,* Vol. 19, pp. 1405-1428

[13] Lucas, Robert E., Jr., 1984, "Money in a Theory of Finance", *Carnegie-Rochester Conference Series on Public Policy,* Vol. 21, pp. 9-46

[14] Lucas, Robert E., Jr. and Stokey, Nancy L., 1983, "Optimal Fiscal and Monetary Policy in an Economy without Capital", *Journal of Monetary Economics,* Vol. 12, pp. 55-93

[15] Lucas, Robert E., Jr. and Stokey, Nancy L., 1987, "Money and Interest in a Cash-in-Advance Economy", *Econometrica,* Vol. 55, No. 9, pp. 491-513

[16] Roubini, Nouriel and Sala-i-Martin, Xavier, 1992, "Financial Repression and Economic Growth", *Journal of Development Economics,* Vol. 39, pp. 5-30

[17] Sargent, Thomas J., 1987, Dynamic Macroeconomic Theory, Harvard University Press

[18] Stockman, Alan C., 1981, "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy", *Journal of Monetary Economics,* Vol. 8, pp. 387-393

[19] Tobin, James, 1965, "Money and Economic Growth", *Econometrica,* Vol. 33, No. 4, pp. 671-84

[20] Valentinyi, Akos, 1995, "How Financial Development and Inflation May Affect Growth", University of Southampton, mimeo

Chapter 4

The Role of Central Bank Operating Procedures in an Economy with Productive Government Spending[1]

4.1 Introduction

It is widely recognized that the central bank operating procedures affect the fluctuations of economic variables. Monetary authorities may control either monetary aggregates or nominal interest rates, but not both independently. Therefore, the monetary authority must decide whether to use the money growth rate or the nominal interest rate as the policy instrument by taking into account that one monetary policy instrument may lead to a superior performance than the other. In this chapter we want to reexamine some of the issues related to the choice of the monetary policy instrument in a general equilibrium model with endogenous growth in which the government spending is financed by means of issuing currency. We evaluate the performance of both monetary instruments from the point of view of fluctuations of endogenous variables and welfare.

As it was already analyzed by Poole (1970) in an IS-LM framework, the problem of finding the optimal policy instrument is irrelevant when we are dealing with a non-stochastic economy. This question only arises under the presence of uncertainty. Poole (1970) evaluates the choice between the two instruments by looking at the output

[1]The content of this chapter is a joint paper with Jordi Caballé.

fluctuations. He finds that, when the origin of disturbances comes mainly from the money demand, to target the nominal interest rate is the best policy in terms of output stabilization, whereas to target the money growth rate is the most stabilizing policy when the origin of disturbances comes mainly from the technology. Concerning prices, he recommends the nominal interest rate targeting as a price stabilizing policy.

After Poole's contribution the question of the optimal choice of the monetary policy instruments was discussed in the literature using more sophisticated frameworks. For example, Carlstrom and Fuerst (1995) consider a cash-in-advance model with portfolio rigidity in the households' cash savings choice, and find that the interest rate targeting is the instrument that outperforms the monetary aggregate targeting in terms of welfare even if the former delivers more volatile output. Collard, Dellas and Ertz (1998) evaluate the two targeting procedures in a growth model with labor augmenting technological progress, and consider the effects of technology, money demand and fiscal shocks. They conclude that the nominal interest rate targeting results in higher welfare and lower volatility of both output and inflation rate regardless of the shocks that are in the origin of the disturbances. Canzoneri and Dellas (1998) study the effect of the choice of the monetary policy instrument on the level of the risk premium in a cash-in-advance economy without capital, and with and without labor contracts inducing rigidities. They find that, under the nominal interest rate targeting, the average level of real interest rate is higher and the prices are less volatile. Nevertheless, they conclude that it is not clear which policy performs better in terms of welfare.

In this chapter we deal with an endogenous growth model in which two kinds of shocks are present: technology and money demand shocks. Technology shocks enter directly into the production function. Money demand shocks are introduced in the form of a modified cash-in-advance constraint, following the approach of Woodford (1991). We evaluate the influence of both shocks on the growth rate of the economy, on the inflation rate, on the income velocity of money and on consumption when the monetary authority follows either the monetary aggregate or the nominal interest rate targeting.

We will consider a production function with government spending, like in Barro (1990). However, Barro (1990) assumes that government spending is financed by a flat rate income tax only. Blackburn and Hung (1996) assume instead that the government finances its spending through seignorage, that is, by printing money. This assumption allows the model to create a new link between monetary shocks and output since the inflationary revenues obtained by the government are transformed into productive spending. We will assume that the government obtains revenues both from taxes and from seignorage. If the revenues from income taxes were disregarded, the model would display unrealistic values both of the money growth rate and of the nominal interest rate and, therefore, we combine both forms of financing as in Palivos and Yip (1995). Needless to say, in equilibrium our model reduces to an AK model as discussed in chapter 4 of Barro and Sala-i-Martin (1995).

Our analysis leads to the following conclusions. Both the growth rate and the money velocity are less volatile under the monetary aggregate targeting, regardless of the shocks that are in the origin of the disturbances. The inflation rate is less volatile under the nominal interest rate targeting. Even if consumption is clearly less volatile under the nominal interest rate targeting, none of the analyzed targeting procedures yields unambiguously higher welfare levels in our economy.

The remainder of the chapter is organized as follows. The model is described in section 4.2. We present the solution technique in section 4.3. We calibrate the model to the US economy and perform a steady state analysis in section 4.4. Results for both targeting procedures are presented in section 4.5. Section 4.6 concludes the chapter.

4.2 The Model

4.2.1 The Households

Let us consider an economy populated by infinitely lived identical households. The preferences of a representative household at time t are given by the following utility function defined over the random stream of consumption $\{c_j\}_{j=t}^{\infty}$:

$$E_t\left[\sum_{j=t}^{\infty}\beta^{j-t}\left(\frac{c_j^{1-\theta}-1}{1-\theta}\right)\right], \tag{4.1}$$

where $\beta \in (0,1)$ is the discount factor and the parameter $\theta > 0$ is the inverse of the elasticity of intertemporal substitution.

Before any trading takes place, agents in every period t learn the state of the economy (A_t, s_t), where A_t is a technology shock and s_t is the money demand shock. We assume that the two shocks are mutually independent. Moreover, the nominal monetary holdings of agents are augmented by a lump sum transfer X_t. Then, the securities market opens. During the securities trading session each consumer selects his portfolio of currency holdings M_t and one-period nominally denominated pure discount bonds. The nominal price of bonds is B_t and their nominal gross rate of return from t to $t+1$ is equal to R_{t+1}. Such a return is paid when the securities market opens in period $t+1$. The securities trading session is subject to the following budget constraint:

$$\frac{M_t}{p_t} + \frac{B_t}{p_t} \leq w_t + \frac{X_t}{p_t}, \tag{4.2}$$

where p_t is the price of the good and w_t is the beginning of period real wealth. After such a financial exchange, individuals produce, obtain the output y_t, and pay taxes (with a flat-tax rate τ) on such an output. At the end of the period individuals purchase consumption goods and make the investment in private capital. Therefore, the real wealth evolves according to

$$w_{t+1} = \frac{p_t}{p_{t+1}}(1-\tau)y_t + \left(\frac{M_t}{p_{t+1}} - \frac{p_t c_t}{p_{t+1}}\right) + \frac{R_{t+1}B_t}{p_{t+1}} - \frac{p_t}{p_{t+1}} i_t, \tag{4.3}$$

where the first term is the real income, the second term in parenthesis is the excess of monetary holdings after consumption purchases have taken place, the third reflects the return from bond holdings, and i_t is the real investment per capita. All the terms in the right hand side of (4.3) are expressed in terms of period $t+1$ goods. Plugging the evolution of wealth (4.3) into the wealth constraint (4.2), we get the following budget constraint:

$$\frac{p_{t-1}}{p_t}c_{t-1} + \frac{p_{t-1}}{p_t}i_{t-1} + \frac{M_t}{p_t} + \frac{B_t}{p_t} \leq \frac{p_{t-1}}{p_t}(1-\tau)y_{t-1} + \frac{M_{t-1}}{p_t} + \frac{R_t B_{t-1}}{p_t} + \frac{X_t}{p_t}. \tag{4.4}$$

The law of motion for private capital is

$$k_{t+1} = i_t + (1-\delta)k_t$$

where δ is the depreciation rate.

In strict cash-in-advance models with uncertainty and a single consumption good, consumers must purchase such a good by using only currency, and the income earned in the current period cannot be converted into money until the next financial exchange. However, following Woodford (1991) we will assume here that a fraction of the t period after tax income can be used for current period purchases. Moreover, like Canzoneri and Dellas (1998) we allow this fraction to fluctuate randomly. Therefore, the cash-in-advance constraint becomes

$$c_t \leq \frac{M_t}{p_t} + s_t(1-\tau)y_t \tag{4.5}$$

where s_t is the money demand shock. Money demand shocks are assumed to be log-normally distributed and to follow an autoregressive process,

$$\ln s_{t+1} = (1-\rho_s)\ln \bar{s} + \rho_s \ln s_t + \varepsilon_{s,t+1}$$

with $\rho_s \in (0,1)$, and where $\ln \bar{s}$ is the unconditional expected value of the logarithm of the money demand shock, and the variables $\varepsilon_{s,t}$ are identically and independently distributed with $\varepsilon_{s,t} \sim N(0,\sigma_s^2)$.[2] The shock s_t can be viewed as a measure of the efficiency of the payment system. Depending on both the after-tax income $(1-\tau)y_t$ and the efficiency of the payment system s_t in the current period, agents know how severe is their cash-in-advance constraint. If we set $s_t = 0$, we would obtain the cash-in-advance constraint usually found in standard monetary models. In this case, only the currency held at the end of the financial exchange could be used to purchase goods. The more general cash-in-advance constraint (4.5) allows a fraction s_t of the after-tax income in period t to be spent immediately, whereas the fraction $(1-s_t)$ cannot be spent until after the financial exchange of the next period. A higher value of s_t corresponds to a more efficient payment system and, obviously, it is associated with a higher velocity of money.

A representative household chooses the stochastic vector sequence $\{c_t, k_{t+1}, M_t, B_t\}_{t=1}^{\infty}$ in order to maximize the expected discounted sum of utility (4.1) subject to the budget constraint (4.4) and the cash in advance constraint (4.5).

4.2.2 The Firms

In this economy there are identical firms, and each of them produces the single good of this economy according to the technology represented by the production function

$$y_t = A_t k_t^{\alpha} g_t^{1-\alpha} \tag{4.6}$$

where y_t is the output per worker, A_t is a random variable that represents the technology shock, k_t is the stock of capital per worker, g_t is the government expenditure per capita and $\alpha \in (0,1)$ is the elasticity of output with respect to private capital. Note that, for the sake of simplicity we are implicitly making simultaneous the payment of taxes, its conversion in public spending, and the production of output by means of

[2] Note that $E(\ln s_t) = \ln \bar{s}$, whereas the conditional expectation is $E_t(\ln s_{t+1}) = (1-\rho_s)\ln \bar{s} + \rho_s \ln s_t$.

the services provided by public spending. Technology shocks are also assumed to be lognormally distributed and to follow an autoregressive process,

$$\ln A_{t+1} = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_t + \varepsilon_{A,t+1},$$

where $\rho_A \in (0, 1)$, $\ln \bar{A}$ is the unconditional expected value of the logarithm of the technology shock and the variables $\varepsilon_{A,t}$ are identically and independently distributed with $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$. We are thus considering just a stochastic version of the model of Barro (1990) in which the flow of government spending is a productive input subject to congestion since it is the spending per capita and not the aggregate spending that enters in the production function. Firms do not pay for the use of the public services accruing from the government spending. We will assume that each firm is owned by a consumer.[3]

4.2.3 The Government

The role of the government in this economy is to set both the fiscal and the monetary policy parameters. The tax rate τ on income is set constant for all periods. Concerning the monetary policy, government has at its disposal two monetary instruments: it can regulate either the monetary aggregate or the nominal interest rate. When the monetary aggregate is targeted, the nominal interest rate is determined endogenously whereas when the nominal interest rate is used as a monetary policy instrument, it is the quantity of money to be printed that accommodates the demand of monetary balances.

We assume that government expenditures are financed by the flat-rate income tax and by printing money (seignorage) so that the net supply of bonds is zero even if the returns of such bonds is potentially set by the government. Therefore, the government

[3] We could assume instead that consumers lend both capital and labor to the firms and that both the rental price r_t of capital and the real wage ϖ_t are set competitively so that firms end up getting zero profits in equilibrium. Therefore,

$$r_t = A_t \alpha k_t^{\alpha-1} g_t^{1-\alpha} \tag{4.7}$$

and

$$\varpi_t = A_t (1 - \alpha) k_t^{\alpha} g_t^{1-\alpha}. \tag{4.8}$$

Obviously, we obtain that the income that individuals get is $y_t = \varpi_t + r_t k_t = A_t k_t^{\alpha} g_t^{1-\alpha}$.

budget constraint is

$$g_t = \tau y_t + \frac{X_t}{p_t}. \tag{4.9}$$

4.2.4 Equilibrium

Let λ_t and η_t be the non-negative Lagrange multipliers associated with the budget (4.4) and the cash-in-advance (4.5) constraints, respectively. The equations that characterize the equilibrium are the following first order conditions, which are obtained from replacing y_t by the production function (4.6), and by taking derivatives of the corresponding Lagrangian with respect to consumption, money, bonds and capital:

$$c_t^{-\theta} = \beta E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right) + \eta_t, \tag{4.10}$$

$$\frac{\lambda_t}{p_t} = \frac{\eta_t}{p_t} + \beta E_t\left(\frac{\lambda_{t+1}}{p_{t+1}}\right), \tag{4.11}$$

$$\frac{\lambda_t}{R_{t+1}} = \beta E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right), \tag{4.12}$$

$$\beta E_t\left(\lambda_{t+2}\frac{p_{t+1}}{p_{t+2}}\left[\alpha(1-\tau)A_{t+1}\left(\frac{g_{t+1}}{k_{t+1}}\right)^{1-\alpha} + (1-\delta)\right]\right) +$$

$$E_t\left(\eta_{t+1}s_{t+1}\alpha(1-\tau)A_{t+1}\left(\frac{g_{t+1}}{k_{t+1}}\right)^{1-\alpha}\right) = E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right), \tag{4.13}$$

and the following transversality conditions:

$$\lim_{j\to\infty} E_t\left(\beta^{t+j}\lambda_{t+j}k_{t+j+1}\right) = 0, \tag{4.14}$$

$$\lim_{j\to\infty} E_t\left(\beta^{t+j}\lambda_{t+j}\frac{B_{t+j}}{p_{t+j}}\right) = 0. \tag{4.15}$$

Combining (4.10) and (4.11), we get that the marginal utility of consumption equals the marginal utility of wealth, λ_t, that is, wealth at the beginning of each period can be converted into consumption regardless of the asset in which this wealth is accumulated.

This is a consequence of having the possibility of acquiring money before consumption takes place. The left hand side of the first order condition on nominal balances (4.11) can be interpreted as the loss of utility due to the acquisition of an extra unit of money. At the margin this amount must be equal to the value of the liquidity service provided by such a unit of money plus the discounted expected utility increase (or decrease) due to capital gains (or losses) resulting from price level changes. Conditions (4.12) and (4.13) combine the costs and expected gains of investing one marginal unit of wealth into bonds and capital, respectively.

Definition: Given the set of initial conditions k_1, M_0, B_0, p_1 the equilibrium consists of stochastic processes $\{c_t, k_{t+1}, i_t, M_t, B_t, R_{t+1}, \mu_t, p_t, g_t\}_{t=1}^{\infty}$ such that

(a) a representative household is maximizing the discounted expected utility (4.1) subject to the budget constraint (4.4) and the cash-in-advance constraint (4.5),

(b) markets for goods, money and bonds clear in every period

$$\begin{aligned} c_t + k_{t+1} - (1-\delta)\, k_t &= (1-\tau) A_t k_t^{\alpha} {g_t}^{1-\alpha}, && (4.16) \\ M_t &= \mu_t M_{t-1}, && (4.17) \\ B_t &= 0, && (4.18) \end{aligned}$$

(c) government runs a balanced budget constraint

$$g_t = \tau A_t k_t^{\alpha} g_t^{1-\alpha} + \frac{X_t}{p_t} \qquad (4.19)$$

where $X_t = (\mu_t - 1)\, M_{t-1}$, and

(d.1) if the government pegs the rate of monetary growth $\mu_t = \mu$ for $t = 1, 2, ...$, where μ is given, or

(d.2) if the government pegs the nominal interest rate, $R_{t+1} = R$ for $t = 1, 2, ...$, where R is given.

4.3 Solution Method

In order to be able to analyze the equilibrium of the economy we have just described, we have to solve the system formed by the equations characterizing such an equilibrium. That is, we need to solve simultaneously the first order conditions (4.10)-(4.13), the market equilibrium equations (4.16)-(4.18), the government budget constraint (4.19), the cash-in-advance constraint (4.5) and the transversality conditions (4.14)-(4.15).

The model does not admit a closed form solution and, therefore, we will apply a numerical technique. The technique applied here will be the one of Uhlig (1997) which is based on the log-linearization of the necessary equations characterizing the equilibrium around steady state. In fact, Uhlig's method is an Euler equation based approach which allows to solve for the recursive equilibrium law of motion using the method of undetermined coefficients. We proceed as follows:

1) We transform the equilibrium equations into a stationary form, expressing variables in ratios,

$$\hat{k}_{t+1}=\frac{k_{t+1}}{k_t},\ \hat{c}_t=\frac{c_t}{k_t},\ \widehat{i}_t=\frac{i_t}{k_t},\ \hat{y}_t=\frac{y_t}{k_t},\ \hat{g}_t=\frac{g_t}{k_t},\ \hat{m}_t=\frac{M_t}{p_t\,k_t},\ f_{t+1}=\frac{p_{t+1}}{p_t},\ \hat{\lambda}_t=\lambda_t k_t^{\theta}$$

where $\hat{k}_{t+1}$, $\hat{c}_t$, $\widehat{i}_t$, $\hat{y}_t$, $\hat{g}_t$, $\hat{m}_t$, f_{t+1} and $\hat{\lambda}_t$ are the growth rate of capital, consumption to capital ratio, investment to capital ratio, output to capital ratio, government spending to capital ratio, real balances to capital ratio, inflation rate and transformed marginal utility of wealth, respectively. After such a transformation, the equilibrium conditions become

$$\begin{aligned}
\hat{c}_t+\widehat{i}_t &= (1-\tau)\hat{y}_t, & (4.20)\\
\hat{k}_{t+1} &= \widehat{i}_t+(1-\delta), & (4.21)\\
\hat{y}_t &= A_t\hat{g}_t^{1-\alpha} & (4.22)\\
\hat{c}_t &= \hat{m}_t+s_t\,(1-\tau)\,\hat{y}_t, & (4.23)\\
\hat{g}_t &= \tau\hat{y}_t+\frac{(\mu_t-1)}{\mu_t}\hat{m}_t, & (4.24)\\
\hat{c}_t^{-\theta} &= \hat{\lambda}_t, & (4.25)
\end{aligned}$$

$$E_t\left(\frac{\hat{\lambda}_{t+1}}{f_{t+1}}\right)=E_t\left\{\beta\frac{\hat{\lambda}_{t+2}}{f_{t+2}}\hat{k}_{t+2}^{-\theta}\left[\alpha\,(1-\tau)\,\hat{y}_{t+1}\,(1-s_{t+1})+1-\delta\right]+\hat{\lambda}_{t+1}s_{t+1}\alpha\,(1-\tau)\,\hat{y}_{t+1}\right\}, \tag{4.26}$$

$$\begin{aligned}
E_t\left(\frac{\hat{\lambda}_t}{R_{t+1}}-\beta\frac{\hat{\lambda}_{t+1}}{f_{t+1}}\hat{k}_{t+1}^{-\theta}\right) &= 0, & (4.27)\\
E_t\left(f_{t+1}-\mu_{t+1}\frac{\hat{m}_t}{\hat{m}_{t+1}}\frac{1}{\hat{k}_{t+1}}\right) &= 0, & (4.28)
\end{aligned}$$

where (4.24) is obtained plugging (4.17) into (4.19), and (4.28) is in fact the money market equilibrium equation (4.17). The remaining equations are self-explanatory.

2) We assign values to parameters and calculate the steady state.

3) We log-linearize the system of equilibrium equations (expressed in ratios) around its steady state. A variable with a *tilde* denotes the log-deviation of such a variable from its steady state,

$$\tilde{h}_t = \ln \hat{h}_t - \ln \bar{h},$$

where $\hat{h}_t$ is a variable to be log-linearized and $\bar{h}$ is its steady state value. It is convenient to write the log-linearized system of equilibrium equations in matrix form. To do so, we define $\tilde{x}_t$ as a $q-$dimensional vector of endogenous state variables, $\tilde{u}_t$ as a $n-$dimensional vector of control variables, and $\tilde{z}_t$ as a $r-$dimensional vector of exogenous state variables. We can then write

$$\begin{aligned} 0 &= A\tilde{x}_t + B\tilde{x}_{t-1} + C\tilde{u}_t + D\tilde{z}_t, \\ 0 &= E_t\left(F\tilde{x}_{t+1} + G\tilde{x}_t + H\tilde{x}_{t-1} + J\tilde{u}_{t+1} + K\tilde{u}_t + L\tilde{z}_{t+1} + M\tilde{z}_t\right), \\ \tilde{z}_{t+1} &= N\tilde{z}_t + \varepsilon_{t+1}, \qquad \text{with } E_t(\varepsilon_{t+1}) = 0 \end{aligned} \tag{4.29}$$

where it is assumed that C is of dimension $l \times n$, with $l \geq n$ and $rank(C) = n$, where l is the number of deterministic equations, F is of dimension $(q + n - l) \times n$, and N is of dimension $r \times r$.

If we consider our original model, there are 2 endogenous state variables, k_t and M_t (since $B_t = 0$ for all periods, we do not take the variable B_t into account), 7 control variables, c_t, λ_t, i_t, y_t, g_t, p_t, and R_{t+1} or μ_t, depending on the monetary policy, and 2 exogenous state variables, A_t and s_t. After applying the transformation no more endogenous state variables appear since $\hat{k}_{t+1}$ depends only on the exogenous shocks, and $\hat{m}_t$ is a function of exogenous shocks and endogenous prices. We thus have $l = 6$ deterministic equations and $n = 9$ control variables. Therefore $l < n$, and the matrix C would not be properly defined. To deal with this problem we redefine some control variables as endogenous state variables, so that q is raised and n is reduced until $l = n$. Therefore, we need to redefine 3 control variables as endogenous state variables to get $q = 3$ and $l = n = 6$.

4) We obtain the recursive equilibrium law of motion in the form

$$\begin{aligned} \tilde{x}_t &= \mathbf{P}\tilde{x}_{t-1} + \mathbf{Q}\tilde{z}_t, \\ \tilde{u}_t &= \mathbf{R}\tilde{x}_{t-1} + \mathbf{S}\tilde{z}_t \end{aligned} \tag{4.30}$$

where the algorithm looks for matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ so that the equilibrium described by these rules is stable. Uhlig (1997) proves that $\mathbf{P}$ is the solution of the following quadratic matrix equation

$$\Psi\mathbf{P}^2 - \Gamma\mathbf{P} - \Theta = 0,$$

where

$$\begin{aligned}\Psi &= F - JC^{-1}A,\\ \Gamma &= JC^{-1}B - G + KC^{-1}A,\\ \Theta &= KC^{-1}B - H,\end{aligned}$$

$\mathbf{R}$ is given by

$$\mathbf{R} = -C^{-1}\left(A\mathbf{P} + B\right),$$

and $\mathbf{Q}$ satisfies

$$\left(N' \otimes \left(F - JC^{-1}A\right) + I_r \otimes \left(J\mathbf{R} + F\mathbf{P} + G - KC^{-1}A\right)\right)\ vec\left(\mathbf{Q}\right) =$$

$$vec\left(\left(JC^{-1}D - L\right)N + KC^{-1}D - M\right),$$

where $\otimes$ is the Kronecker product, I_r is the identity matrix of size $r \times r$, and $vec(\cdot)$ denotes columnwise vectorization, and $\mathbf{S}$ is given by [4]

$$\mathbf{S} = -C^{-1}\left(A\mathbf{Q} + D\right).$$

[4]Harald Uhlig's Matlab programs are used to solve for the recursive law of motion. They are available at http://cwis.kub.nl/~few5/center/STAFF/uhlig/toolkit.dir/toolkit.htm.

4.4 Steady State Analysis

4.4.1 Calibration

We calibrate the model to match the quarterly US data by following Collard, Dellas and Ertz (1998). To this aim, we rewrite the equilibrium equations of the transformed model (4.20)-(4.28) in a non-stochastic (with $\sigma_A = \sigma_s = 0$) steady state as follows:

$$\begin{aligned}
\bar{c} + \bar{\imath} &= (1-\tau)\bar{y}, && (4.31)\\
\bar{k} &= \bar{\imath} + (1-\delta), && (4.32)\\
\bar{y} &= \bar{A}\bar{g}^{1-\alpha}, && (4.33)\\
\bar{c} &= \bar{m} + \bar{s}(1-\tau)\bar{y}, && (4.34)\\
\bar{g} &= \tau\,\bar{y} + \frac{(\bar{\mu}-1)}{\bar{\mu}}\bar{m}, && (4.35)\\
\bar{c}^{-\theta} &= \bar{\lambda} && (4.36)\\
1 &= \beta\bar{k}^{-\theta}\left\{\alpha(1-\tau)\bar{y}\left[1+\left(\bar{R}-1\right)\bar{s}\right]+(1-\delta)\right\}, && (4.37)\\
\bar{f} &= \beta\bar{R}\bar{k}^{-\theta} && (4.38)\\
\bar{k}\bar{f} &= \bar{\mu}, && (4.39)
\end{aligned}$$

where the variables with a *bar* denote their steady state values. The value of the steady state growth rate $\bar{k}$ is set to 1.007 (0.7% quarterly corresponding to the annual growth rate of 2.8%). As reported in Cooley and Prescott (1995), the steady state ratio between investment and capital is 0.076 and thus we set $\bar{\imath} = 0.076$ and calculate δ for annual data. The corresponding depreciation rate for quarterly data is $\delta = 0.017$. Parameters α and τ are set to the reasonable values, $\alpha = 0.36$ and $\tau = 0.33$. The steady state money growth rate is $\bar{\mu} = 1.0081$. The discount factor β is adjusted with the parameter θ. When the values of θ are 0.5, 1 and 2, we use 0.987, 0.9902 and 0.997 as values of β, respectively.

Finally, we assign the following parameters for the stochastic processes: $\bar{A} = 1$, $\rho_A = 0.955$, $\sigma_A = 0.0075$ and $\bar{s} = 0.4$, $\rho_s = 0.9482$, $\sigma_s = 0.0567$. The benchmark parameters are summarized in the following table:

Benchmark parameters:
$\bar{\mu} = 1.0081$ ($\bar{R} = 1.018$)
$\bar{A} = 1$, $\rho_A = 0.955$, $\sigma_A = 0.0075$
$\bar{s} = 0.4$, $\rho_s = 0.9482$, $\sigma_s = 0.0567$
$\alpha = 0.36$
$\delta = 0.017$
$\tau = 0.33$
$\theta = 0.5, 1, 2$ ($\beta = 0.987, 0.9902, 0.997$)

Table 4.1: Values of the parameters

4.4.2 Steady State Effects of the Monetary Policy

We will first analyze the effect of the monetary policy on the steady state growth rate of the non-stochastic version of the model. In our model an increase in government spending translates into an increase in income. Therefore, when more money is injected into the economy, the government obtains more revenues from seignorage and, thus, output increases. As expected, we clearly observe a Tobin effect, that is, a raise in the money growth rate increases both the capital accumulation and the growth rate of output. The magnitude of such an increase becomes smaller as the value of the parameter θ increases since then the elasticity of intertemporal substitution decreases and individuals want to smooth their consumption paths. Higher money growth rate increases inflation as would suggest the equation (4.39) alone. The increase is higher for higher values of θ because for higher θ the growth rate exhibits less change.

When the nominal interest rate is targeted, we observe that an increase in the nominal interest rate delivers a decrease in the growth rate and an increase in inflation. The pattern we observe in the growth rate is the same as in the case of the monetary aggregate targeting. As the money growth rate adjusts to the changes in the nominal interest rate, prices behave in the same way for different values of θ.

The reaction of the growth rate of the economy and the inflation rate to the changes in the money growth rate and the nominal interest rate are plotted in Figure 4.1.

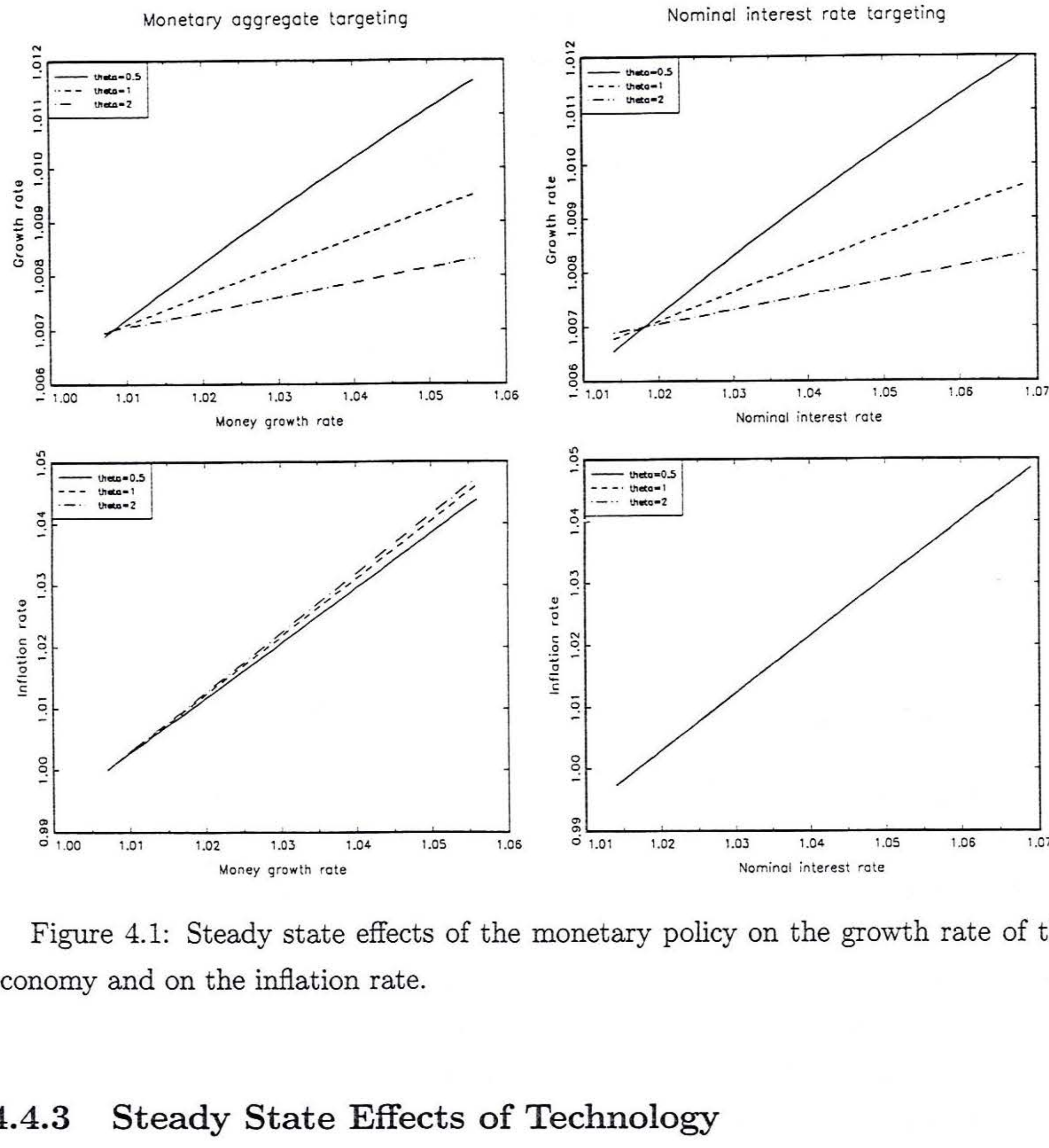

Figure 4.1: Steady state effects of the monetary policy on the growth rate of the economy and on the inflation rate.

4.4.3 Steady State Effects of Technology

We want to see now what is the effect of a change in $\bar{A}$ both on the steady state growth rate and on the inflation rate. When the steady state level of the technology increases, it has a positive effect on the growth rate because a higher $\bar{A}$ corresponds to higher output and the increase in output produces more tax revenues, which in turn increase the output even more trough government spending. This implies an increase in consumption, which makes agents increase their demand for real balances and, therefore, prices must decrease in equilibrium. We thus observe an increase in the growth rate and a decrease in the inflation rate for both monetary policy instruments. The results are plotted in Figure 4.2. As it can be seen, the patterns for different values of θ are similar to the case discussed in section 4.4.2.

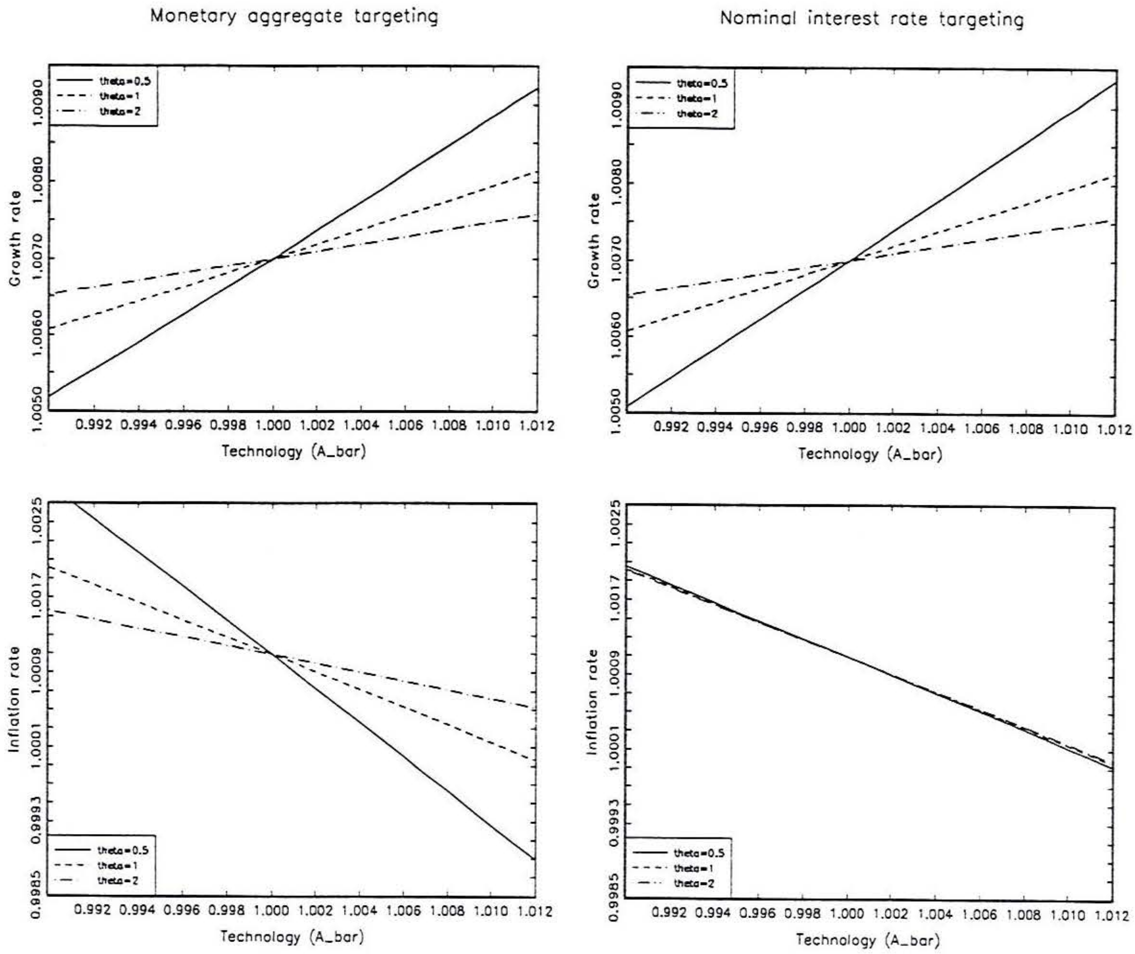

Figure 4.2: Steady state effects of the technology on the growth rate of the economy and on the inflation rate.

4.4.4 Steady State Effects of the Efficiency of the Payment System

When $\bar{s}$ increases, a higher fraction of the individuals' current period income can be used to purchase consumption goods and the demand of real monetary balances is thus smaller. When the monetary aggregate is used as the monetary instrument real balances can be reduced only by an increase in prices so that inflation goes up. Such a decrease in real balances translates into less seignorage by the government. Therefore, both output and capital accumulation are reduced as a consequence of the decrease in government spending. The resulting decrease in growth rates is thus achieved.

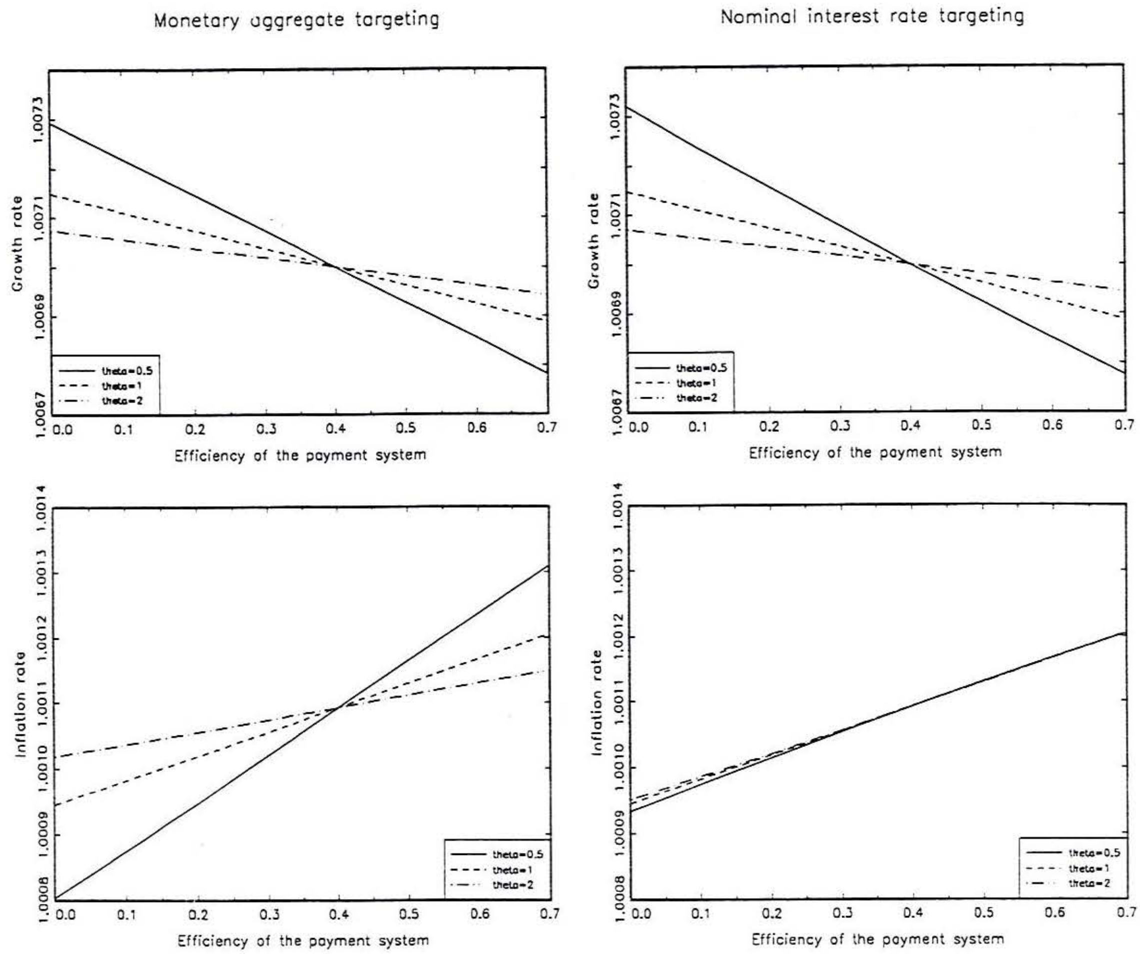

Figure 4.3: Steady state effects of the efficiency of the payment system on the growth rate of the economy and the inflation rate.

When the nominal interest rate is fixed, real balances are negatively related with the nominal interest rate since the latter is the opportunity cost of holding money. This translates into less revenues accruing from seignorage and less growth. As expected inflation reacts positively to the increase in the nominal interest rate.

The behavior of the steady state growth rate and the inflation rate for both monetary policy instruments is plotted in Figure 4.3.

4.5 Effects of the Two Targeting Procedures on the Stochastic Equilibrium

4.5.1 Monetary Aggregate Targeting

We consider here that the monetary policy consists of pegging a constant money growth rate, $\mu_t = \mu$ for all periods. We solve the system (4.20)-(4.28) using the numerical technique described in section 4.3. The log-linearized equations and the matrices of endogenous state variables, control variables and exogenous state variables are stated in Appendix 1. Note that, according with section 4.3, we consider as state variables $\hat{k}_t$, $\hat{\lambda}_{1,t}$ (which is defined to be equal to $\hat{\lambda}_{t+1}$), $f_{2,t}$ (which is defined to be equal to f_t), R_t and $f_{1,t}$ (which is equal to f_{t+1}), whereas the control variables are $\hat{c}_t$, $\hat{\lambda}_{2,t}$ (which is defined to be equal to $\hat{\lambda}_t$), $\widehat{\imath_t}$, $\hat{y}_t$, $\hat{m}_t$ and $\hat{g}_t$.

To see the effect of each shock we will consider a reaction of the economy to a technology shock and to a money demand shock separately. We will assume that the economy is in the non-stochastic steady state at time $t = 0$. At time $t = 1$ the perturbation $\varepsilon_{A,t}$ ($\varepsilon_{s,t}$) is selected in such a way that the technology factor (the money demand shock) experiences a 1% deviation from the steady state. Such perturbations become $\varepsilon_{A,t} = 0$ and $\varepsilon_{s,t} = 0$, respectively, for all $t > 1$. We analyze the impulse-responses to both shocks to determine the effect they have on the growth rate and inflation rate. In Figures 4.4 and 4.5 we plot the impulse-responses to a technology and a money demand shock, respectively, for the following variables: growth rate of capital, output to capital ratio and the inflation rate.

A positive technology shock increases output directly through A_t, equation (4.6), and indirectly through the government spending, equation (4.9). Government spending increases because (i) the tax revenues increase, and (ii) the revenues from seignorage increase due to a decrease in prices. Both consumption and investment increase, because there is available more income which can be optimally distributed between them. Since the growth of nominal money supply is fixed and consumption increases, the real money demand also increases and, hence, prices fall to clear the markets. Because of the increase in investment, capital accumulation is accelerated. Both higher capital accumulation and higher government spending imply an increase in the growth rate of the economy.

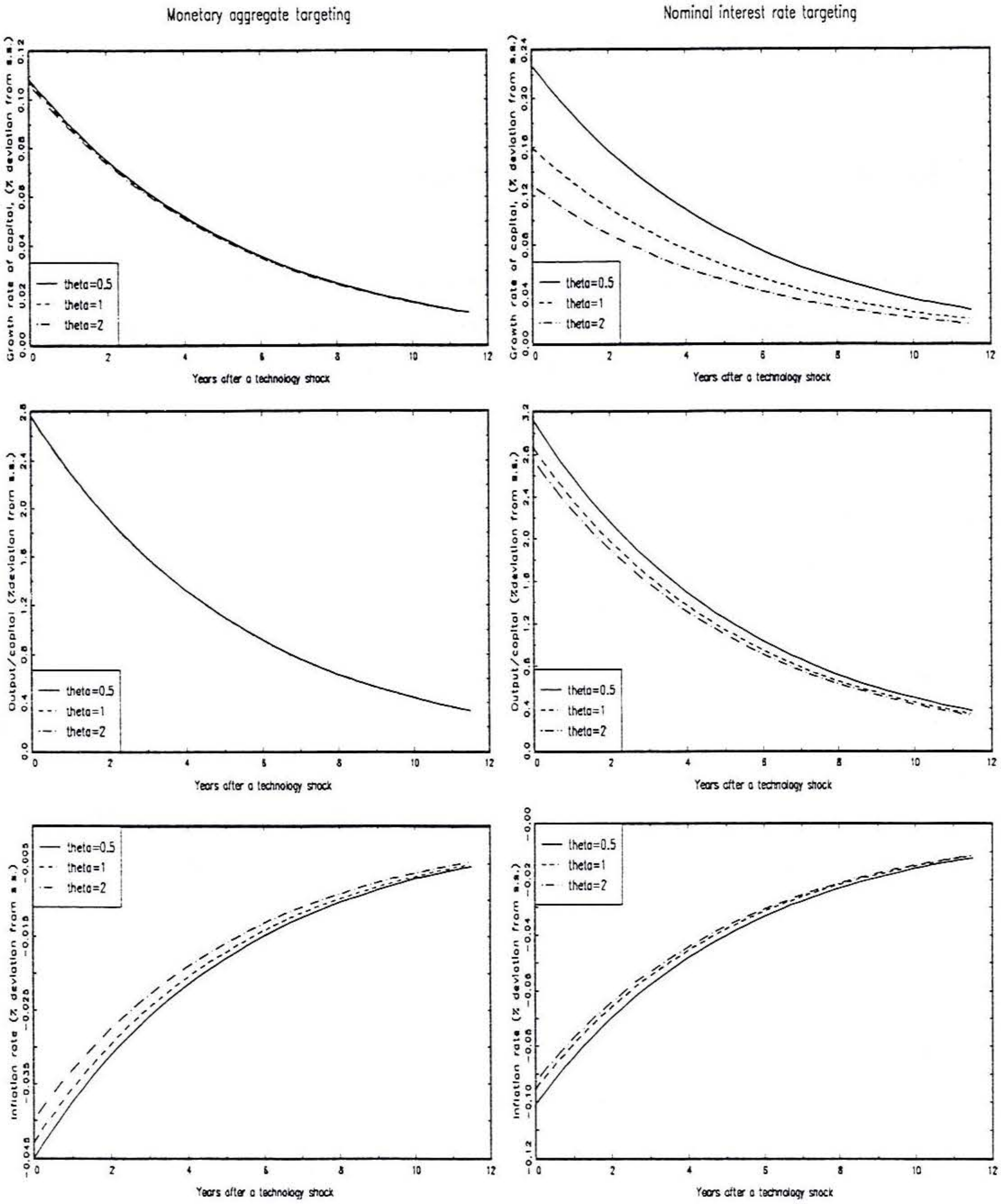

Figure 4.4: Impulse-responses of the growth rate of capital, output to capital ratio, and the inflation rate to a technology shock, for the monetary aggregate targeting (left column) and for the interest rate targeting (right column).

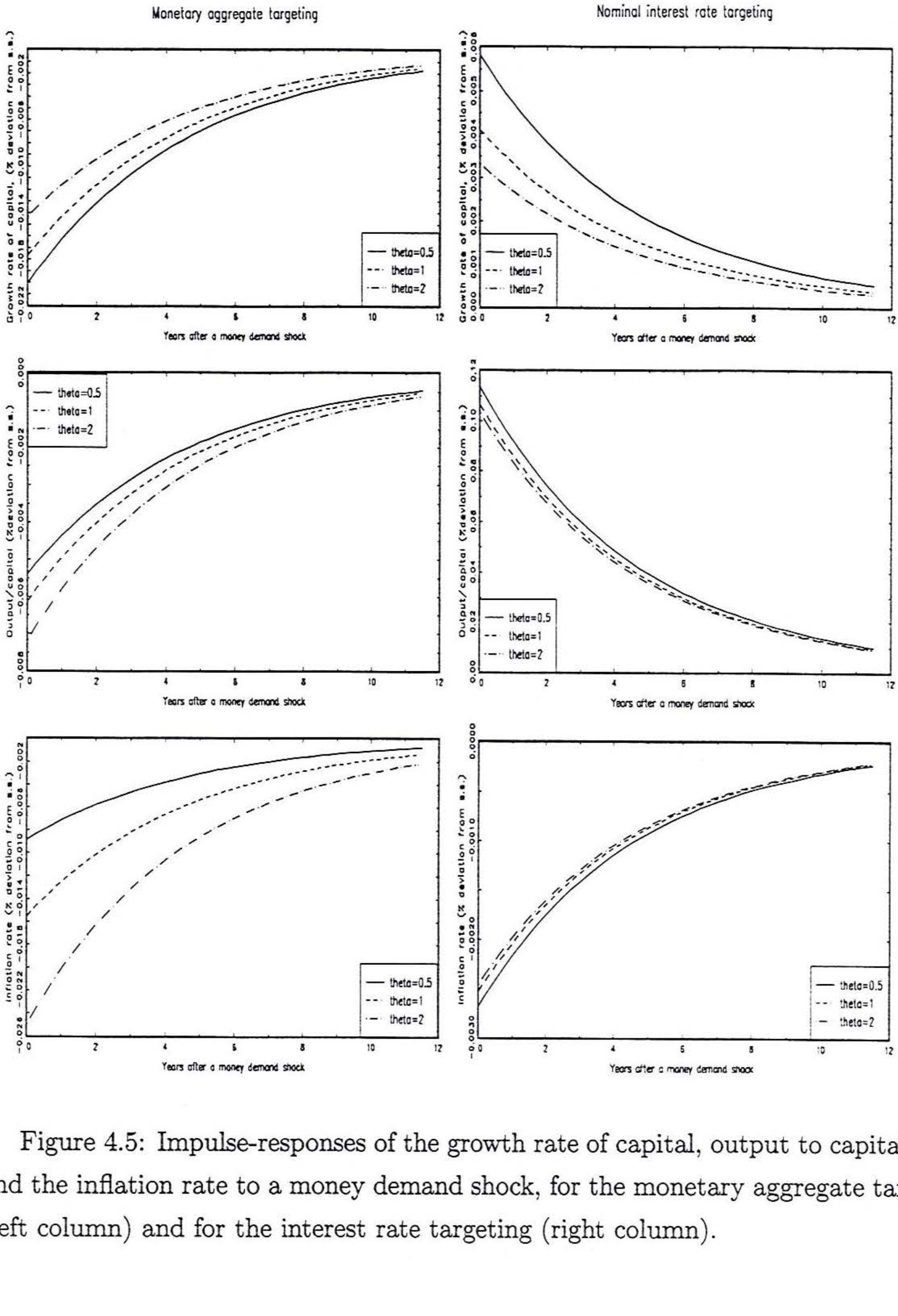

Figure 4.5: Impulse-responses of the growth rate of capital, output to capital ratio, and the inflation rate to a money demand shock, for the monetary aggregate targeting (left column) and for the interest rate targeting (right column).

A positive money demand shock relaxes the liquidity constraint and, therefore, the value of money will be lower. This implies less government spending since the seignorage will be smaller. As a result, (adjusted) income goes down and the growth rate becomes smaller in the short run. Since the rate of monetary growth is fixed, the reduction in the (adjusted) monetary balances is achieved through a change in current prices. However since the shock is transitory, the value of money in the future will be higher than in the current period as the monetary system becomes again more inefficient. This means that future prices will be lower than the current ones. This amounts thus to a reduction in the short run inflation rate. Note that this transitory effect on the inflation rate is the opposite to the one obtained when we analyzed the permanent effects in the non-stochastic stationary equilibrium.

Finally, note that the behavior of the impulse-responses is qualitatively similar for different values of the inverse of the elasticity of intertemporal substitution θ.

4.5.2 Nominal Interest Rate Targeting

We consider now that the monetary policy target is to peg a constant nominal interest rate, $R_{t+1} = R$ for all periods. To fix the nominal interest rate, the government must let the money supply respond to both the technology and the money demand shocks. To solve the model we conjecture a solution for prices of the form

$$p_t = \kappa_t \lambda_t \tag{4.40}$$

where κ_t is a non-stochastic proportionality factor which is potentially time-dependent. Plugging (4.40) into the first order condition on bonds (4.12), we get the evolution of the proportionality factor

$$\frac{\kappa_{t+1}}{\kappa_t} = \beta R. \tag{4.41}$$

We plug (4.41) into the system (4.20)-(4.28) and solve the corresponding equilibrium equations. The log-linearized equations and the vectors of state and control variables are stated in Appendix 2. In order to apply Uhlig's method we consider as state variables $\hat{k}_t$, μ_t, and f_t, whereas the control variables are now $\hat{c}_t$, $\hat{\lambda}_t$, $\widehat{i}_t$, $\hat{y}_t$, $\hat{m}_t$ and $\hat{g}_t$.

To see the effect of each shock separately we again analyze the impulse-responses of the growth rate of capital, output to capital ratio and the inflation rate. A positive transitory technology shock increases output and consequently consumption and investment grow faster. Such an increase of the growth rate must be accompanied by a reduction in the inflation rate so as to increase the real monetary balances needed for purchasing consumption goods at a faster pace.

A positive transitory money demand shock means that future seignorage will be higher since money will have more value. Therefore, thanks to the government spending, the future output will be higher than the current one. This means that the growth rate increases. On the other hand, since the monetary system will be less efficient in the future, real monetary balances will have higher value in the future periods. This means that future prices will be lower and therefore, inflation goes down. Note that the effects of a shock in the money demand are the opposite in sign to the ones corresponding to the permanent shocks in the non-stochastic steady state.

Impulse-responses to technology and money demand shocks for $\hat{k}_{t+1}$, $\hat{y}_t$, and f_t are plotted in Figures 4.4 and 4.5.

4.5.3 Comparison of the Two Targeting Procedures

In this section we compare the performance of the targeting procedures from the point of view of fluctuations of consumption, growth rate and inflation rate. We also make some welfare considerations.

Fluctuations

Technology and money demand shocks are channelled into the behavior of all variables in a different way under the monetary aggregate and nominal interest rate targets. We define the volatility of a variable h_t, σ_h, as a standard error of the prediction of h_t, i.e., as the standard error of the residuals of the regression

$$\ln h_t = \zeta_h + \rho_h \ln h_{t-1} + \varepsilon_{h,t}. \tag{4.42}$$

We evaluate the volatility of consumption, growth rate, inflation rate and income velocity (which is defined as the ratio of nominal output to nominal balances, $v_t = \frac{y_t p_t}{M_t}$) for different values of the parameter θ.

		$\mu_t = \mu$ for all t			$R_{t+1} = R$ for all t		
	θ	(A, s)	(A)	(s)	(A, s)	(A)	(s)
σ_c	0.5	0.024	0.0168	0.0177	0.007	0.0068	0.0011
	0.7	0.024	0.0168	0.0167	0.0097	0.0095	0.0015
	0.9	0.023	0.0168	0.0159	0.0114	0.00112	0.0018
	1	0.023	0.0169	0.0155	0.012	0.0118	0.0019
	1.5	0.022	0.0169	0.0137	0.013	0.01358	0.0023
	2	0.021	0.017	0.0124	0.015	0.01449	0.0024
σ_{y_{t+1}/y_t}	0.5	0.021	0.0207	0.0009	0.0228	0.0225	0.0004
	0.7	0.0209	0.0207	0.0008	0.0221	0.0217	0.0039
	0.9	0.0209	0.0207	0.0007	0.0217	0.0213	0.0039
	1	0.0209	0.0207	0.0007	0.0216	0.0212	0.0038
	1.5	0.0209	0.0207	0.0006	0.0212	0.0208	0.0038
	2	0.0209	0.0207	0.0005	0.0210	0.0207	0.0037
σ_f	0.5	0.00063	0.00035	0.00052	0.0008	0.00078	0.000157
	0.7	0.00076	0.00034	0.00069	0.00077	0.00076	0.000152
	0.9	0.0009	0.00034	0.00084	0.00076	0.00074	0.000149
	1	0.00097	0.00033	0.00091	0.00075	0.00074	0.000148
	1.5	0.00125	0.00032	0.00012	0.00074	0.00073	0.000145
	2	0.00149	0.00031	0.00146	0.00073	0.000722	0.000144
σ_v	0.5	0.02204	0.0215	0.0044	0.02471	0.0224	0.0104
	0.7	0.02204	0.0215	0.0046	0.02428	0.0218	0.0103
	0.9	0.02210	0.0215	0.0049	0.02408	0.0216	0.0101
	1	0.02220	0.0215	0.005	0.02403	0.0216	0.0099
	1.5	0.02226	0.0215	0.0055	0.02382	0.0214	0.0098
	2	0.02244	0.0215	0.006	0.02378	0.0214	0.0098

Table 4.2: Volatility of consumption, growth rate, inflation rate, and income velocity for the two targeting procedures; μ – money growth rate, R – nominal interest rate, θ – inverse of the elasticity of intertemporal substitution, (A, s) – both shocks operate, (A) – only technology shocks operate, (s) – only money demand shocks operate, σ_c – volatility of consumption, σ_{y_{t+1}/y_t} – volatility of the growth rate, σ_f – volatility of the inflation rate, σ_v – volatility of the income velocity.

As can be seen from Table 4.2, consumption is much less volatile under the nominal interest rate targeting, growth rate of the economy and income velocity are a little bit less volatile under the monetary aggregate targeting, and inflation rate is less volatile under the nominal interest rate targeting.[5] When the monetary aggregate is targeted, both technology and money demand shocks contribute in a similar magnitude to the fluctuations in consumption. Fluctuations of the growth rate and income velocity are caused mostly by disturbances in technology. Money demand shocks are the ones that contribute more to the volatility of the inflation rate. When the nominal interest rate is targeted, consumption, growth rate and inflation rate vary mostly due to disturbances in technology. Notice that the contribution of the money demand shocks to the fluctuations of the income velocity is rather low.

We see that the nominal interest rate targeting accommodates better the effect of the money demand shocks on prices than the monetary aggregate targeting. This is caused by the fact that, under the fixed nominal interest rate, money supply can react to a money demand shock.

Welfare

If the government cares about the welfare of the agents, its objective should be to maximize the following welfare function:

$$W = E_t \left(\sum_{j=t}^{\infty} \beta^{j-t} \, \frac{c_j^{1-\theta} - 1}{1-\theta} \right). \tag{4.43}$$

To ensure that the expectation in (4.43) is finite, we write the consumption as

$$c_t = c_t^* \bar{k}^t,$$

where c_t^* is the detrended consumption, and rewrite the welfare function as

$$W = E_t \left(\sum_{j=t}^{\infty} \left[(\beta^*)^{j-t} \, \frac{\left(c_j^*\right)^{1-\theta}}{1-\theta} - \frac{\beta^{j-t}}{1-\theta} \right] \right)$$

where $\beta^* = \beta \bar{k}^{1-\theta}$ must be strictly less than 1 so as to ensure the desired convergence. Such an inequality is satisfied by our calibration of the model. We calculate the welfare

[5] Volatility of a variable h_t (where h_t in this case stands for consumption, growth rate, inflation rate and money velocity) reported in the table is calculated as an average value of σ_h obtained for 500 shock realizations, where σ_h is calculated using (4.42).

		$\mu_t = \mu$	for all t
θ	(A, s)	(A)	(s)
0.5	−96.69 (−101.62,−91.75)	−97, 18 (−100.63,−93.72)	−97.12 (−100.40,−93.83)
0.7	−127.63 (−135.82,−119.45)	−127.62 (−133.79,−121.44)	−127.75 (−132.97,−122.52)
0.9	−171.55 (−185.14,−157.96)	−171.25 (−181.72,−160.78)	−171.47 (−179.91,−163.03)
1	−199.95 (−217.29,−182.61)	−200.73 (−213.71,−187.76)	−200.56 (−211.46,−189.65)
1.5	−464.5 (−529.6,−399.5)	−460.15 (−512.1,−408.2)	−460.1 (−496.8,−423.3)
2	−1171.3 (−1413.6,−929.0)	−1166.0 (−1357.7,−974.3)	−1152.6 (−1283.7,−1021.5)

Table 4.3: Welfare levels achieved under the monetary aggregate targeting (confidence intervals in the brackets); μ – money growth rate, R – nominal interest rate, θ – inverse of the elasticity of intertemporal substitution, (A, s) – both shocks operate, (A) – only technology shocks operate, (s) – only money demand shocks operate.

as an empirical mean of 1000 shock realizations of time series with a horizon of 1000 periods.

We let the origin of disturbances be both technology and money demand shocks, only technology and only money demand shocks, respectively, and evaluate the welfare. As can be seen from Tables 4.3 and 4.4, the confidence intervals for welfare under different targeting procedures almost completely overlap in all cases. Even if the volatility of consumption is lower for the nominal interest rate targeting, we are unable to choose the monetary policy instrument that clearly achieves higher welfare. Therefore, there is no central bank procedure that clearly dominates the other neither when considering the shocks to operate separately, nor both at the same time.

θ	$R_{t+1} = R$ for all t		
	(A, s)	(A)	(s)
0.5	−96.47 (−103.19,−89.76)	−96.84 (−103.18,−90.50)	−97.05 (−98.18,−95.92)
0.7	−127.43 (−137.09,−117.77)	−127.25 (−136.79,−117.72)	−127.83 (−129.44,−126.23)
0.9	−171.37 (−186.14,−156.61)	−171.09 (−185.73,−156.45)	−171.38 (−173.85,−168.90)
1	−200.20 (−218.35,−182.04)	−201.06 (−218.58,−183.53)	−199.61 (−202.76−196.45)
1.5	−463.0 (−526.4,−399.6)	−461.7 (−524.9,−398.5)	−457.2 (−467.6,−446.6)
2	−1166.3 (−1392.6,−940.1)	−1175.5 (−1398.0,−953.0)	−1128.2 (−1166.0,−1090.4)

Table 4.3: Welfare levels achieved under the nominal interest rate targeting (confidence intervals in the brackets); μ – money growth rate, R – nominal interest rate, θ – inverse of the elasticity of intertemporal substitution, (A, s) – both shocks operate, (A) – only technology shocks operate, (s) – only money demand shocks operate.

4.6 Conclusion

In this chapter we have analyzed the effects of two targeting objectives of monetary policy. We have performed two kinds of analyses: the one concerning the non-stochastic version of the economy and the one concerning its stochastic counterpart. In a non-stochastic economy we have seen the effects of permanent changes in the technology and the efficiency of the payment system (the money demand) on the steady state growth rate and the inflation rate. In a stochastic economy we have studied how the mentioned variables react to unexpected shocks in technology and the efficiency of the payment system and what is the contribution of particular shocks to the fluctuations of the growth rate, inflation rate, money velocity, and consumption.

Both the non-stochastic and the stochastic economies behave generally in an analogous way when taking into account the shocks in the technology, no matter if they are permanent or transitory. This is not the case when we consider changes in the efficiency of the payment system. The basic difference we observe is caused by the different future value of money when changes are transitory or permanent. When a positive transitory shock to the efficiency of the payment system occurs, the value of money after experiencing a decrease returns to its original level. Nevertheless, when a permanent increase in the efficiency of the payment system occurs, the value of money in the future decreases to a new level. The different behavior exhibited by prices is reflected in different reactions of the growth rate and the inflation rate, respectively.

Concerning the comparison of the targeting procedures with respect to fluctuations, we find that output is less volatile under the monetary aggregate targeting, regardless of the origin of disturbances. Poole (1970) arrives to the same result when the origin of disturbances are the shocks in money demand, but not when they are the shocks in technology. Concerning the inflation rate, our results confirm the findings of Poole (1970), Canzoneri and Dellas (1998) and Collard, Dellas and Ertz (1998), that is, the inflation rate is less volatile under the interest rate targeting. Surprisingly, shocks in the efficiency of the payment system are not very influential in determining the volatility of the income velocity of money, neither under the monetary aggregate nor under the nominal interest rate targeting. The ambiguous result concerning welfare makes us conclude that none of the analyzed targeting procedures is clearly superior if the goal of the government is to maximize the individuals' expected lifetime utility.

One question that has not been analyzed is what would be the effect of shocks on the endogenous variables if the government allowed for more inflation and thus for higher seignorage. We already know that, concerning the long run accumulation of capital, the Tobin effect holds when the steady state money growth rate or the nominal interest rate increase. However, we do not know the magnitude in which the particular shocks contribute to the fluctuations of the variables under different levels of the two monetary instruments.

We have studied the behavior of the economy for passive monetary policies.[6] Another analysis that remains to be done is to evaluate the performance of the economy under active monetary policies. This amounts to analyzing how should the monetary authorities react to current disturbances in the technology and the efficiency of the payment system in order to stabilize growth rate (or to achieve higher welfare), when they face the choice of regulating either the monetary aggregates and or the nominal interest rates.

[6] Analogously to Poole (1970) a passive monetary policy is understood as the one that follows a target that is fixed for all periods, i.e., constant money growth rate and constant nominal interest rate. An active monetary policy would be a function of lagged responses to the disturbances and policy actions.

4.7 Appendix

Appendix 1: Monetary Aggregate Targeting

Assuming $\mu_t = \mu$ for all t, the log-linearized system (4.20)-(4.27) can be written in the following way:

$$\begin{aligned}
\bar{c}\tilde{c}_t + \widetilde{\bar{i}i}_t &= (1-\tau)\,\bar{y}\tilde{y}_t, && (4.44)\\
\bar{k}\tilde{k}_{t+1} &= \widetilde{\bar{i}i}_t, && (4.45)\\
\tilde{y}_t &= \tilde{A}_t + (1-\alpha)\tilde{g}_t, && (4.46)\\
\bar{c}\tilde{c}_t &= \bar{m}\tilde{m}_t + \bar{s}\bar{y}\,(1-\tau)\,\tilde{s}_t + \bar{s}\bar{y}\,(1-\tau)\,\tilde{y}_t, && (4.47)\\
\bar{g}\tilde{g}_t &= \tau\bar{y}\tilde{y}_t + \frac{(\mu-1)}{\mu}\bar{m}\tilde{m}_t, && (4.48)\\
-\theta\tilde{c}_t &= \tilde{\lambda}_{2,t}, && (4.49)
\end{aligned}$$

$$\begin{aligned}
0 &= E_t\left(\frac{\beta\bar{k}^{-\theta}}{\bar{f}}\left[\alpha\,(1-\tau)\,\bar{y}\,(1-\bar{s})+1-\delta\right]\left\{-\theta\tilde{k}_{t+2}+\tilde{\lambda}_{1,t+1}-\tilde{f}_{1,t+1}-\tilde{\lambda}_{2,t+1}\right\}\right.\\
&\quad + \left[\frac{1}{\bar{f}}-\frac{\beta\bar{k}^{-\theta}}{\bar{f}}\,(1-\delta)\right]\tilde{y}_{t+1} + \left[-\frac{\beta\bar{k}^{-\theta}}{\bar{f}}\alpha\,(1-\tau)\,\bar{y}\bar{s}+\bar{s}\alpha\,(1-\tau)\,\bar{y}\right]\tilde{s}_{t+1}\\
&\quad \left. + \frac{1}{\bar{f}}\,\tilde{f}_{2,t+1}\right), && (4.50)
\end{aligned}$$

$$0 = E_t\left(\tilde{\lambda}_{2,t+1}-\tilde{\lambda}_{1,t}\right), \qquad (4.51)$$

$$0 = E_t\left(\tilde{m}_{t+1}-\tilde{k}_{t+1}+\tilde{f}_{2,t+1}-\tilde{m}_t\right), \qquad (4.52)$$

$$0 = E_t\left(\tilde{f}_{2,t+1}-\tilde{\lambda}_{2,t+1}-\tilde{R}_{t+1}+\theta\tilde{k}_{t+1}+\tilde{\lambda}_{2,t}\right), \qquad (4.53)$$

$$0 = E_t\left(\tilde{f}_{2,t+1}-\tilde{f}_{1,t}\right). \qquad (4.54)$$

where $\tilde{\lambda}_{1,t+1} = \tilde{\lambda}_{t+2}$, $\tilde{\lambda}_{2,t+1} = \tilde{\lambda}_{t+1}$ and $\tilde{f}_{1,t+1} = \tilde{f}_{t+2}$, $\tilde{f}_{2,t+1} = \tilde{f}_{t+1}$. We define these new variables because we want to transform a system of the third order difference equations into a one of the second order difference equations.

The resulting recursive rules have the following form:

$$\begin{pmatrix} \tilde{k}_{t+1} \\ \tilde{\lambda}_{1,t} \\ \tilde{f}_{2,t} \\ \tilde{R}_{t+1} \\ \tilde{f}_{1,t} \end{pmatrix} = \mathbf{P} \begin{pmatrix} \tilde{k}_t \\ \tilde{\lambda}_{1,t-1} \\ \tilde{f}_{2,t-1} \\ \tilde{R}_t \\ \tilde{f}_{1,t-1} \end{pmatrix} + \mathbf{Q} \begin{pmatrix} \tilde{A}_t \\ \tilde{s}_t \end{pmatrix},$$

$$\begin{pmatrix} \tilde{c}_t \\ \tilde{\lambda}_{2,t} \\ \tilde{i}_t \\ \tilde{y}_t \\ \tilde{m}_t \\ \tilde{g}_t \end{pmatrix} = \mathbf{R} \begin{pmatrix} \tilde{k}_t \\ \tilde{\lambda}_{1,t-1} \\ \tilde{f}_{2,t-1} \\ \tilde{R}_t \\ \tilde{f}_{1,t-1} \end{pmatrix} + \mathbf{S} \begin{pmatrix} \tilde{A}_t \\ \tilde{s}_t \end{pmatrix}.$$

Appendix 2: Nominal Interest Rate Targeting

We assume that $R_{t+1} = R$ for all t and log-linearize the system (4.20)-(4.28) around its steady state

$$\begin{aligned}
\bar{c}\tilde{c}_t + \bar{i}\tilde{i}_t &= (1-\tau)\,\bar{y}\tilde{y}_t, && (4.55)\\
\bar{k}\tilde{k}_{t+1} &= \bar{i}\tilde{i}_t, && (4.56)\\
\tilde{y}_t &= \tilde{A}_t + (1-\alpha)\tilde{g}_t, && (4.57)\\
\bar{c}\tilde{c}_t &= \bar{m}\tilde{m}_t + \bar{s}\bar{y}\,(1-\tau)\,\tilde{s}_t + \bar{s}\bar{y}\,(1-\tau)\,\tilde{y}_t && (4.58)\\
\bar{g}\tilde{g}_t &= \tau\bar{y}\tilde{y}_t + \frac{\bar{\mu}-1}{\bar{\mu}}\bar{m}\tilde{m}_t + \frac{\bar{m}}{\bar{\mu}}\tilde{\mu}_t, && (4.59)\\
-\theta\tilde{c}_t &= \tilde{\lambda}_t, && (4.60)
\end{aligned}$$

$$0 = E_t\left(-\tilde{\mu}_{t+1} + \tilde{m}_{t+1} + \tilde{k}_{t+1} + \tilde{f}_{t+1} - \tilde{m}_t\right), \quad (4.61)$$

$$\begin{aligned}
0 = {} & E_t\Big(\tilde{\lambda}_{t+1} - \theta\tilde{k}_{t+1} + \beta\bar{k}^{-\theta}\alpha\,(1-\tau)\,\bar{y}\,[1+(R-1)\bar{s}]\;\tilde{y}_{t+1} + \\
& \beta\bar{k}^{-\theta}\alpha\,(1-\tau)\,\bar{y}(R-1)\bar{s}\;\tilde{s}_{t+1}\Big), \quad (4.62)
\end{aligned}$$

$$0 = E_t\left(\tilde{\lambda}_{t+1} - \theta\tilde{k}_{t+1} - \tilde{f}_{t+1} - \tilde{\lambda}_t\right). \quad (4.63)$$

The recursive rules have the following form:

$$\begin{pmatrix} \tilde{k}_{t+1} \\ \tilde{\mu}_t \\ \tilde{f}_t \end{pmatrix} = \mathbf{P}\begin{pmatrix} \tilde{k}_t \\ \tilde{\mu}_{t-1} \\ \tilde{f}_{t-1} \end{pmatrix} + \mathbf{Q}\begin{pmatrix} \tilde{A}_t \\ \tilde{s}_t \end{pmatrix},$$

$$\begin{pmatrix} \tilde{c}_t \\ \tilde{\lambda}_t \\ \tilde{i}_t \\ \tilde{y}_t \\ \tilde{m}_t \\ \tilde{g}_t \end{pmatrix} = \mathbf{R}\begin{pmatrix} \tilde{k}_t \\ \tilde{\mu}_{t-1} \\ \tilde{f}_{t-1} \end{pmatrix} + \mathbf{S}\begin{pmatrix} \tilde{A}_t \\ \tilde{s}_t \end{pmatrix}.$$

Bibliography

[1] Barro, J. Robert, 1990, "Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy,* Vol. 98, No. 5, pp. S103-S125

[2] Barro, J. Robert and Sala-i-Martin, Xavier, 1995, Economic Growth, McGraw-Hill

[3] Blackburn, Keith and Hung, Victor T. Y., 1996, "Equilibrium Growth in a Monetary Economy", *Seoul Journal of Economics,* Vol. 9, No. 2, pp. 145-161

[4] Canzoneri, Matthew B. and Dellas, Harris, 1998, "Real Interest Rates and Central Bank Operating Procedures", *Journal of Monetary Economics,* Vol. 42, pp. 471-494

[5] Carlstrom, Charles T. and Fuerst, Timothy S., 1995, "Interest Rate Rules vs. Money Growth Rules A Welfare Comparison in a Cash-in-Advance Economy", *Journal of Monetary Economics,* Vol. 36, pp. 247-267

[6] Cole, Harold L. and Ohanian, Lee E., 1997, "Shrinking Money and Monetary Business Cycles", Federal Reserve Bank of Minneapolis, Research Department Working Paper 579

[7] Collard, Fabrice, Dellas, Harris and Ertz, Guy, 1998, "Poole Revisited", IRES, Universitē Catolique de Louvain, Discussion Paper 9823

[8] Cooley, Thomas F. and Prescott, Edward C., 1995, "Economic Growth and Business Cycles", in Frontier of Business Cycle Research, Ed. Thomas F. Cooley, Princeton University Press

[9] Den Haan, Wouter J. and Marcet, Albert, (1990), "Solving the Stochastic Growth Model by Parameterizing Expectations", *Journal of Business and Economic Statistics,* Vol. 8, No. 1, pp. 31-34

[10] Jones, Larry E. and Manuelli, Rodolfo E., 1995, "Growth and the Effects of Inflation" *Journal of Economic Dynamic and Control,* Vol. 19, pp. 1405-1428

[11] Lucas, Robert E. Jr., 1980, "Equilibrium in a Pure Currency Economy", *Economic Inquiry,* Vol. 18, pp. 203-220

[12] Palivos, Theodore and Yip, Chong K., 1995, "Government Expenditure Financing in an Endogenous Growth Model: A comparison", *Journal of Money, Credit and Banking,* Vol. 27, No. 4, pp. 1159-1178

[13] Poole, Wiliam, 1970, "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model", *Quarterly Journal of Economics,* Vol. 84, pp.197-216

[14] Uhlig, Harald, 1997, "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily", CentER, University of Tilburg and CEPR, mimeo

[15] Woodford, Michael, 1991, "Currency Competition and the Transition to Monetary Union: Does Competition Between Currencies Lead to Price Level and Exchange Rate Stability?" in European Financial Integration, Eds. Alberto Giovannini and Colin Mayer, Cambridge University Press